1. (15 points) A uniform plane wave traveling in free space is expressed as($\varepsilon_0 = 10^{-9}/(36\pi)$ F/m and $\mu_0 = 4\pi\times10^{-7}$ H/m)

$$\bar{E}(x,t) = 5\sin(\omega t - k_0 x - \pi/6)\hat{y} + 6\sin(\omega t - k_0 x + \pi/4)\hat{z}, \quad \text{(unit: V/m)} \qquad (1)$$

where ω and k_0 are the angular frequency of the wave and wavenumber in free space.

(a) What is the propagation direction and the free space intrinsic impedance? (2 points)

(b) Please express (1) in the phasor form. (3 points)

(c) Please express the solution of magnetic field, $\bar{H}(x,t)$? (4 points)

(d) Find the time-average power density of the wave in W/m^2. (6 points)
(Show detailed derivations)

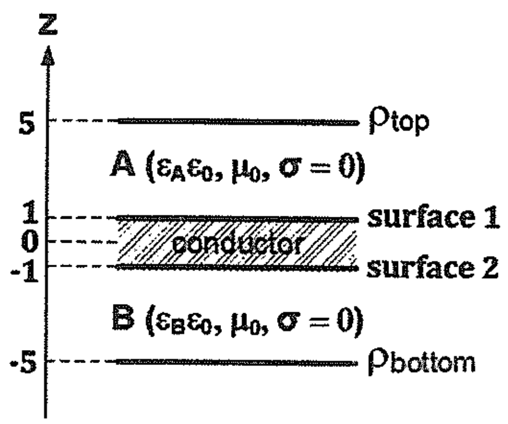
2. (20 points) Consider two infinite planar conducting sheets at $z = \pm 5$ cm, which create a vertically polarized electric field $E_0\hat{z}$ inside the internal area of free space. (Show detailed derivations)

(a) Please find the induced charge densities, ρ_{top} and ρ_{bottom}, at $z = 5$ cm and $z = -5$ cm, respectively. (5points)

(b) Now one inserts an infinite conductor slab of thickness, 2cm, into the internal area at $z=0$ cm to form the configuration below. The free space is also replaced by dielectric materials. Assume the charge densities are ρ_{top} and ρ_{bottom} uniformly distributed over the two charge sheets at $z = \pm 5$ cm. Find the surface charge densities at the two surfaces of the conductor slab. (5 points)

(c) From (b), please find the electrical fields in region A and B, respectively. (5points)

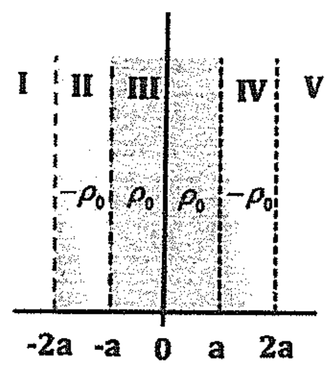
(d) What will happen to (b) and (c) if the conductor slab with a new thickness, 1.5 cm, is moved to $z = 2$ cm inside the internal area ? (5 points)



3. (15 points) Consider the charge distribution given by the densities:

$$\rho = \begin{cases} -\rho_0 & \text{for } -2a < x < -a \\ +\rho_0 & \text{for } -a < x < a \\ -\rho_0 & \text{for } a < x < 2a \end{cases}, \qquad (2)$$

where ρ_0 is a constant. Please find the electrical fields in the five regions.



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4. (15 points) The voltage and current waves in Fig.4(a) can be represented as

$$V(z)=V_+e^{-jkz}+V_-e^{jkz}=V_+e^{-jkz}\left[1+\Gamma(z)\right]$$

$$I(z)=\frac{V_+}{Z_0}e^{-jkz}-\frac{V_-}{Z_0}e^{jkz}=\frac{V_+}{Z_0}e^{-jkz}\left[1-\Gamma(z)\right]$$

where $\Gamma(z)=\Gamma_L e^{2jkz}$ and $\Gamma_L=\frac{V_-}{V_+}=\frac{Z_L-Z_0}{Z_L+Z_0}$

(a) Derive the formula of input impedance $Z_{in}=Z(z=-\ell)$. (5 points)

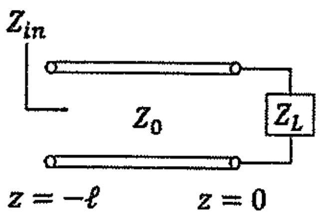

Fig. 4(a)

(b) In Fig.4(b), three transmission lines (each with a short-circuited load) are connected at $z=0$. The lengths of these transmission lines are ℓ_1, ℓ_2 and ℓ_3, respectively; the wavenumbers on these transmission lines are k_1, k_2 and k_3, respectively; and the characteristic impedances of these transmission lines are Z_1, Z_2 and Z_3, respectively. Apply KVL and KCL at $z=0$ to derive the resonant condition. (5 points)

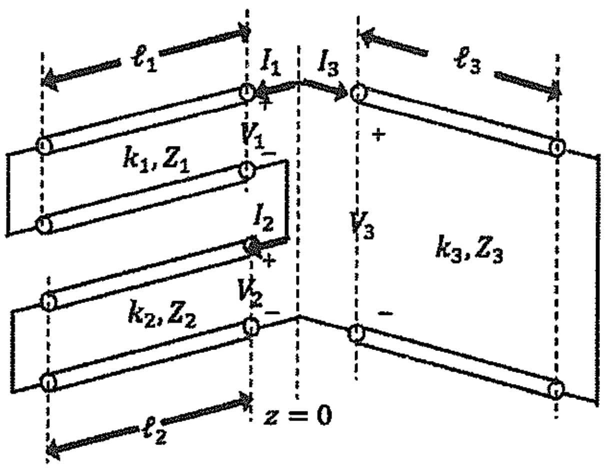

Fig. 4(b)

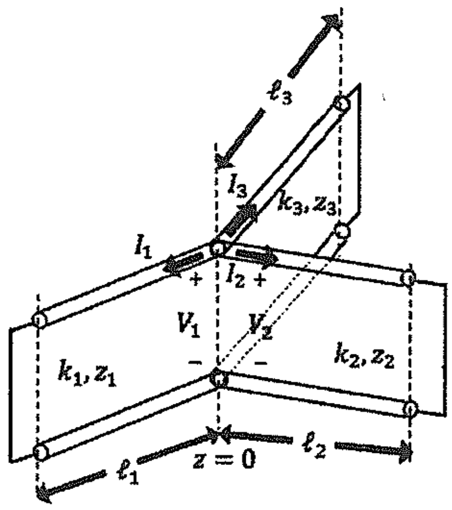

Fig. 4(c)

(c) In Fig.4(c), three transmission lines (each with a short-circuited load) are connected at $z=0$. The lengths of these transmission lines are ℓ_1, ℓ_2 and ℓ_3, respectively; the wavenumbers on these transmission lines are k_1, k_2 and k_3, respectively; and the characteristic impedances of these transmission lines are Z_1, Z_2 and Z_3, respectively. Apply KVL and KCL at $z=0$ to derive the resonant condition (5 points)

5. (15 points) As shown in Figure 5, the magnetic field of an incident plane wave of TM polarization is represented as $\bar{H}_i(\bar{r})=\hat{y}H_0e^{jk_{1x}x-jk_zz}$. The reflected magnetic field can be represented as $\bar{H}_r(\bar{r})=\hat{y}RH_0e^{-jk_{1x}x-jk_zz}$, and the transmitted magnetic field can be represented as $\bar{H}_t(\bar{r})=\hat{y}TH_0e^{jk_{2x}x-jk_zz}$, where R is the reflection coefficient and T is the transmission coefficient.

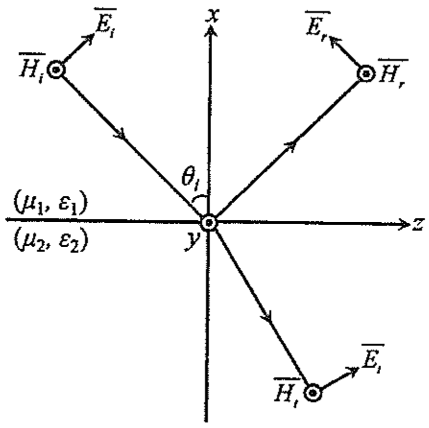

Fig.5

(a) Derive the incident, reflected and transmitted electric fields by using $\overline{E}(\overline{r}) = \frac{1}{j\omega\varepsilon}\nabla\times\overline{H}(\overline{r})$. (5 points)

(b) Impose the boundary condition that the tangential electric and magnetic fields are continuous at $x=0$ to solve R and T. (5 points)

(c) If $\varepsilon_1 > \varepsilon_2$ and $\theta > \theta_c$, ($\theta_i = \theta_c$ is the critical angle) represent R and T as $R = |R|e^{j\psi}$ and $T = |T|e^{j\zeta}$. Derive the expressions of $|R|$, ψ, $|T|$ and ζ. (5 points)

6. (20 points) The vector potential $\overline{A}(\overline{r})$ induced by a Hertzian dipole $\hat{z}I\ell$ at the origin can be represented in the spherical coordinate as $\overline{A}(\overline{r}) = \hat{z}\frac{\mu I\ell}{4\pi r}e^{-jkr}$.

(a) Derive the magnetic field in the spherical coordinate by using $\overline{H}(\overline{r}) = \frac{1}{\mu}\nabla\times\overline{A}(\overline{r})$. (5 points)

(b) Derive the electric field in the spherical coordinate by using $\overline{E}(\overline{r}) = \frac{1}{j\omega\varepsilon}\nabla\times\overline{H}(\overline{r})$. (5 points)

(c) Derive the time-average power density by using the far-field expressions . (5 points)

(d) Derive the directivity as a function of zenith angle θ and azimuth angle ϕ, i.e., the ratio of the power density radiated by the antenna as a function of (θ, ϕ) to the average power density. (5 points)

<Hint> In spherical coordinate, $\nabla\times\overline{A} = \frac{1}{r^2\sin\theta}\begin{vmatrix} \hat{r} & r\hat{\theta} & r\sin\theta\hat{\phi} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial\theta} & \frac{\partial}{\partial\phi} \\ A_r & rA_\theta & r\sin\theta A_\phi \end{vmatrix}$.

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