1. (10%) Suppose we are given the following facts about a sequence $x[n]$:
   - $x[n]$ is periodic with period $N = 6$.
   - $\sum_{n=0}^{5} x[n] = 2$
   - $\sum_{n=2}^{7} (-1)^n x[n] = 1$
   - $x[n]$ has the minimum power per period among the set of signals satisfying the preceding three conditions.

   Please answer the following questions.

   (a) Is it possible to determine $x[n]$ given the above four conditions? (3%)

   (b) If it is possible to determine $x[n]$, please show $x[n]$. If not, please give your reason. (7%)

2. (6%) Let $x(t)$ be a signal whose Fourier transform is

$$X(j\omega) = \delta(\omega) + \delta(\omega - \pi) + \delta(\omega - 5)$$

   and let

$$h(t) = u(t) - u(t-2)$$

   Please answer the following questions.

   (a) Is $x(t)$ periodic? (3%)

   (b) Is $x(t) * h(t)$ periodic? (3%)

3. (6%) Consider a continuous-time LTI system whose frequency response is

$$H(j\omega) = \int_{-\infty}^{\infty} h(t)e^{-j\omega t}dt = \frac{\sin(4\omega)}{\omega}.$$

   If the input to this system is a periodic signal

$$x(t) = \begin{cases} 1 & 0 \le t < 4 \\ -1 & 4 \le t < 8 \end{cases}$$

   with period $T = 8$, determine the corresponding system output $y(t)$.

4. (6%) How many signals have a Laplace transform that may be expressed as

$$\frac{(s-1)}{(s+2)(s+3)(s^2+s+1)}$$

   in its region of convergence?

5. (6%) Let $x[n]$ be a signal whose rational z-transform $X(z)$ contains a pole at $z = 1/2$. Given that

$$x_1[n] = (\frac{1}{4})^n x[n]$$

   is absolutely summable and

$$x_2[n] = (\frac{1}{8})^n x[n]$$

   is not absolutely summable. Determine whether $x[n]$ is left sided, right sided, or two sided.

6. (16%) Determine if the following statements are true or false.
   (a) If $x[n]$ and $y[n]$ are both periodic signals with fundamental period N, $x[n]+y[n]$ is still periodic.
   (b) A linear time-invariant system cannot be non-causal.
   (c) An odd and imaginary signal always has an odd and imaginary Fourier transform.
   (d) The convolution of an odd Fourier transform with an even Fourier transform is always odd.
   (e) If $x[n]=0$ for $n<N_1$ and $h[n]=0$ for $n<N_2$, then $x[n]*h[n]=0$ for $n<N_1+N_2$.
   (f) If $y[n]=x[n]*h[n]$, then $y[n-1]=x[n-1]*h[n-1]$.
   (g) If $y(t)=x(t)*h(t)$, then $y(-t)=x(-t)*h(-t)$.
   (h) If $x(t)=0$ for $t>T_1$ and $h(t)=0$ for $t>T_2$, then $x(t)*h(t)=0$ for $t>T_1+T_2$.

7. (5%) A digital wireless communication system employs 16QAM modulation and operates in a 10MHz (10^7 hertz) band at center frequency 2450 MHz. The roll-off factor of the pulse shaping function is 0.25. The channel code used at the transmitter is a rate 1/2 turbo code. For the transmitter, what is the largest possible number of message **bits** per second can be sent?

8. (10%) Consider a detection problem over a real-valued additive Gaussian noise channel $Y=X+Z$, where $Z\sim\mathcal{N}(0,\sigma^2)$ is a zero-mean Gaussian random variable with variance σ^2. X takes values in a standard 4PAM constellation set uniformly at random, and the average energy per symbol is E_s. The minimum distance detector is used for detection.
   (a) Suppose Gray mapping is used. Compute the average number of **bits** per symbol that are **correctly** detected at the receiver. (5%)
   (b) Suppose the detector reports a special message "`UNKNOWN`" if the ratio of the largest likelihood over the second largest likelihood is smaller than e^C, $C>0$. What is the probability of getting `UNKNOWN` at the receiver? (5%)

9. (15%) For a complex random variable H, we use the notation $H\sim\mathcal{CN}(0,\sigma^2)$ to denote that the real part $\mathrm{Re}\{H\}$ and the imaginary part $\mathrm{Im}\{H\}$ are i.i.d Gaussian $\mathcal{N}(0,\sigma^2/2)$.

   In the following, consider a channel $Y=Hx+Z$, where x is the transmitted symbol using BPSK constellation with E_s being the energy per symbol, $H\sim\mathcal{CN}(0,1)$ is the random channel coefficient, and $Z\sim\mathcal{CN}(0,N_0)$ is the additive Gaussian noise.
   (a) Show that the phase of H is uniformly distributed over $[0,2\pi]$. (3%)
   (b) Suppose the receiver knows the realization of H. Derive the optimal detection rule and the optimal probability of error conditioned on the realization of H, that is, given $H=h$. Please use Q function $\mathrm{Q}(t)\triangleq\int_t^\infty\frac{1}{\sqrt{2\pi}}e^{-u^2/2}\,du$ to express your answer. (4%)
   (c) Suppose the receiver does not know the realization of H. Derive the optimal probability of error. (4%)
   (d) Following (c), the receiver does not know the realization of H. Propose a new binary constellation set that has a better probability of error than Part (c). Give the reason why it is better. (4%)

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10. (20%) Consider a detection problem of a real-valued symbol x given two observations Y_1 and Y_2, where

$$Y_1 = x + Z_1,\ Y_2 = x + Z_2, \quad Z_1 \sim \mathcal{N}(0, \sigma_1^2),\ Z_2 \sim \mathcal{N}(0, \sigma_2^2),\ \sigma_1^2 < \sigma_2^2.$$

Z_1 and Z_2 are jointly Gaussian but not necessarily independent, because the noises might contain some common interference. Let x are chosen uniformly from a 2^ℓ-ary standard PAM constellation set. E_s denotes the average energy per symbol.

(a) Suppose the correlation coefficient between Z_1 and Z_2 is ρ, where $|\rho| \leq 1$. Derive the optimal detection rule. Express your answer as explicitly as possible. (6%)

(b) Let $\rho = 0$ and the constellation set is 4-PAM. Derive the optimal probability of error and use Q function to express your answer. (6%)

(c) A NTU GICE student claims that Y_1 alone can never be a sufficient statistics for detecting x. Is he/she correct? If so, prove it. If not, find the necessary and sufficient condition that Y_1 alone is a sufficient statistics. (8%)

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