國立臺灣大學 107 學年度碩士班招生考試試題

1. (10%) The number of nonnegative integer solutions to

$$L \le x_1 + x_2 + \cdots + x_n \le H$$

is _________.

2. (10%) The solution to the recurrence equation

$$a_n = 2a_{n-1} + 3a_{n-2}$$

with $a_0 = 1$ and $a_1 = 1$ is $a_n =$ _________.

3. (10%) The generating function for the square numbers $1^2, 2^2, 3^2, \ldots$ is _________.

4. (10%) The number of reflexive symmetric relations on A where $|A| = m$ is _________.

5. (10%) The number of simple, labeled graphs (with self-loops allowed) with n nodes is _________.

The number of simple, labeled graphs with n nodes and m edges is _________.

6. (10%) Let $A = \begin{bmatrix} 2 & 2 \\ 1 & 3 \end{bmatrix}$. Please find $f(A)$, where $f(t) = t^4 - 3t^3 - 6t^2 + 7t + 2$.

7. (10%) Let $A = \begin{bmatrix} 0 & 0 & 0 & 0 & a_0 \\ -1 & 0 & 0 & 0 & a_1 \\ 0 & -1 & 0 & 0 & a_2 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & -1 & a_{n-1} \end{bmatrix}_{n\times n}$ and I_n be the $n\times n$ identity matrix. Find $det(A + tI_n)$.

8. (10%) Consider a subspace $V = \mathrm{P}_2(\mathrm{t})$ of $\mathrm{P}(\mathrm{t})$ with inner product defined as

$$\langle f, g \rangle = \int_0^1 f(t) g(t) dt.$$

Please find an orthogonal set of $\{1, t, t^2\}$ with integer coefficients and the projection of t^3 onto V.

9. (10%) Your answer will be considered correct only if all the true statements are selected.
Let A be an $n\times n$ matrix.
(a) If x_1 and x_2 are the eigenvectors of A, then $x_1 + x_2$ is also an eigenvector of A.
(b) If $A^T = -A$, then A is singular.
(c) If $A^2 = A$. $(A + I)^n = I + (2^n + 1)A$.
(d) If $A = A^T$, then A is diagonalizable.
(e) If $A = \begin{bmatrix} B & C \\ O & D \end{bmatrix}$ and B and D are invertible, then $A^{-1} = \begin{bmatrix} B^{-1} & -B^{-1}CD^{-1} \\ O & D^{-1} \end{bmatrix}$.

10. (10%) Your answer will be considered correct only if all the true statements are selected.
(a) Suppose $\{u, v, w\}$ is linearly independent, then $\{u + v, v + w, w + u\}$ is also linearly independent.
(b) If two matrices A and B are similar, then they have the same eigenvalues.
(c) If two matrices A and B are similar, then they have the same eigenvectors.
(d) Let V be a vector space of $m\times n$ matrices over $\mathcal{R}$. $\langle A, B \rangle = tr(B^T A)$ defines an inner product in V.
(e) If U and W are subspaces of a finite-dimensional inner product space V, then $(U + W)^\perp = U^\perp \cup W^\perp$.

試題隨卷繳回