※ 注意：請用 2B 鉛筆作答於答案卡，並先詳閱答案卡上之「畫記說明」。　(禁止使用計算機)

第一大題: 單選題，每題 5 分，共 50 分。

PART I. Multiple Choice Questions (5 points for each question)

1. Which of the following statements is true?
   (A) Dividing both sides of a valid congruence by an integer preserves validity.
   (B) If $(S,\preccurlyeq)$ is a poset and every two elements of S are comparable, then $(S,\preccurlyeq)$ is a well-ordered set.
   (C) There is a polynomial time algorithm to determine if a given integer is a prime.
   (D) Strong induction is more powerful than mathematical induction.
   (E) $C(r,0) - C(r,1) + C(r,2) - \cdots + (-1)^r C(r,r) = 1$.

2. Which of the following statements is true?
   (A) It is possible that a relation R is both reflexive and irreflexive.
   (B) A complete graph is always not a bipartite graphs.
   (C) $\forall x\exists y F(x,y) = \exists y\forall x F(x,y)$.
   (D) $\exists x(F_1(x) \wedge F_2(x)) = \exists x F_1(x) \wedge \exists x F_2(x)$
   (E) $\forall x(F_1(x) \vee F_2(x)) = \forall x F_1(x) \vee \forall x F_2(x)$

3. How many zeros are there at the end of 2018! ?
   (A) 501　(B) 502　(C) 503　(D) 504　(E) 505

4. How many sets in the following are countable?
   - {(x, y): x and y are integers}
   - {(x, y, z): x, y and z are integers}
   - The set of rational numbers
   - The set of all finite strings over a finite alphabet
   - The set of all (valid) Java programs.

   (A) 1　(B) 2　(C) 3　(D) 4　(E) 5

5. What is the time complexity of the following program segment?

```
for (i = 0; i <= n; i++) {
    j = i;
    do j = j div 2 while (j is even);
} /* note that j div 2 returns the integer part of the division result */
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   (A) $\Theta(\log n)$　(B) $\Theta(n)$　(C) $\Theta(n\cdot\log n)$　(D) $\Theta(n^2)$　(E) none of the above

6. Let A denote the set of collision resistant functions, B denote the set of preimage resistant functions, and C denote the set of second preimage resistant functions. Which of the following is correct?
   (A) $A \subseteq B$　(B) $A \subseteq C$　(C) $B \subseteq A$　(D) $B \subseteq C$　(E) $C \subseteq A$

7. Let n be the number of onto functions from a set with 6 elements to a set with 3 elements. Which of the following is true?
   (A) $100 < n \leq 200$　(B) $200 < n \leq 300$　(C) $300 < n \leq 400$　(D) $400 < n \leq 500$　(E) $500 < n \leq 600$

8. What is the value of $2018^{9999} \bmod 13$?
   (A) 1　(B) 3　(C) 5　(D) 7　(E) 9

9. Let n be the number of possible outcomes for a horse race with four horses to finish if ties are possible. Which of the following is true? (Note: Two or three or four horses may tie.)
   (A) $n \leq 40$　(B) $40 < n \leq 50$　(C) $50 < n \leq 60$　(D) $60 < n \leq 70$　(E) $n > 70$

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10. Consider the following functions:

(1) $(1 + 1/n)^n$ (2) $(\log n)!$ (3) $\log (n!)$ (4) 1.0001^n
(5) $2^{\log n \log\log n}$ (6) $(\log n)^{2018}$ (7) 0.9876^{5n} (8) $2^{0.001n}$

How many of them cannot be bounded by a polynomial function?

(A) 2 (B) 3 (C) 4 (D) 5 (E) 6

第二大題: 單選題，每題 10 分，共 50 分。

PART II. Multiple Choice Questions (10 points for each question)

11. Consider the group $H = Z_{19}^*$. Let G be the subgroup of H generated by the element 4. (i.e., $G=\{4^i \bmod 19 \mid 1 \le i \le 18\}$). How many generators does G hvae?

(A) 2 (B) 3 (C) 4 (D) 5 (E) 6

12. Consider the following function:

$$A(m,n) = \begin{cases} 2n & if\ m = 0 \\ 0 & if\ m \ge 1\ and\ n = 0 \\ 2 & if\ m \ge 1\ and\ n = 1 \\ A(m-1, A(m, n-1)) & if\ m \ge 1\ and\ n > 1 \end{cases}$$

Which of the following values is closest to the value of $A(2, 5)$?

(A) 10^{10} (B) 10^{100} (C) 10^{1000} (D) 10^{10000} (E) 10^{20000}

13. How many problems in the following have known polynomial time solutions?

- Find the longest path from a node u to a node v in a weighted graph.
- Given a program P, determine if P contains a virus.
- Given the product of two primes p and q, compute the value of p and q.
- Determine if a given graph is 4-colorable.
- Determine if two given graphs are isomorphic?
- Determine if two given graphs are non-isomorphic?
- Given a graph G, determine if G has a simple path that passes through every vertex exactly once.
- Given a graph G, determine if G has a simple path containing every edge of G.
- Given a Boolean formula, determine if it is satisfiable.
- Given a Boolean formula, determine if it is a tautology.

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

14. Consider relations on the set $\{0,1\}$. Let m, n, k be the numbers of reflexive relations, irreflexive relations and asymmetric relations, respectively. Which of the following is true?

(A) $m+n+k \le 10$ (B) $m+n+k = 11$ (C) $m+n+k = 12$ (D) $m+n+k = 13$ (E) $m+n+k \ge 14$

15. Which of the following statements is not correct?

(A) There are 160 bit strings of length eight either start with a 1 bit or end with the two bits 00.
(B) 9 cards must be selected from a deck of 52 cards to guarantee that at least three cards of the same suit are chosen.
(C) A shelf holds 12 books in a row. There are 56 ways to choose five books so that no two adjacent books are chosen.
(D) There are 10 permutations of 12345 that leave 3 in the third position but leave no other integer in its own position.
(E) There are 78 nonnegative integer solutions for the equation $x_1 + x_2 + x_3 = 11$.

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