

※ Please write down the detailed calculation process.

1. (10%) A one-story storage building is to have a volume of 250,000 cubic feet. The roof costs \$32 per square foot, the walls cost \$10 per square foot, and the floor costs \$8 per square foot. Find the dimensions that minimize the cost of the building.

2. (10%) Consider the double integral

$$\int_0^1 \int_{2x-1}^{\sqrt{x}} 2xydydx.$$

- a. (5%) Rewrite the above double integral by exchanging the order of integration.  
b. (5%) Evaluate the double integral derived in (a).
3. (10%) Find the least squares approximation  $h(x) = ax + ax^2$  for  $f(x) = e^{-x}$ ,  $0 \leq x \leq 1$ . [Hint: Solve  $a$  by minimizing the integral of the squared difference between  $f(x)$  and  $h(x)$ .]

4. (14%) Suppose that you currently have \$40,000 invested in a savings account earning 3% per year compounded continuously. Market conditions are improving and you decide to continuously transfer 15% of the balance of your savings account into a stock fund earning 12% per year. You also withdraw \$5,000 from the savings account each year for expenses.

- a. (10%) Set up and solve a linked pair of differential equations to determine respectively the amount of money in the savings account and stock fund  $t$  years from now.  
b. (4%) How long does it take for the savings account to be exhausted? How much money is in the stock fund at this time point?

5. (6%) If  $\lim_{x \rightarrow 0} (x^{-3} \sin(3x) + rx^{-2} + s) = 0$ , what are the values of  $r$  and  $s$ ?

6. (10%) Find  $\frac{d}{dx} \ln \sec 5x$ .

7. (10%) Evaluate the definite integral  $\int_0^1 \sqrt{4-x^2} x dx$ .

8. (10%) Find the area of an ellipse by using the parametric equations  $x = 5 \cos \theta$  and  $y = 6 \sin \theta$ .

9. (10%) Find the interval of convergence for the power series  $x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \dots$ .

10. (10%) Solve the differential equation  $x^2 dy + 2xy dx + 2x dx + dy = 0$ .