1. Check the following incompressible flow for continuity and determine the vorticity. (20%)

$u = \frac{y^3}{2} + 2x - x^2y;\quad v = xy^2 - 2y - \frac{x^3}{2};\quad w = 6$

2. Water is pumped through a pipeline to a treatment plant at a rate of 30 cfs. The 5-ft-diameter suction line is 3000 ft long, and the 3-ft-dimeter discharge line is 1500 ft long. The pump adds energy to the water at a rate of 40 ft lb/lb and the total head loss is 56 ft. If the water pressure at the pipeline entrance is 50 psi, what is the pressure at the exit, which is 15 ft higher than the entrance? (20%)

3. A viscous oil flows steadily between two stationary horizontal and parallel plates. The flow is laminar and fully developed. The total gap width between the plates is h = 10 mm. The oil viscosity is 0.5 N•s/m^2, and density is 1200 kg/m^3. The pressure gradient is -1000 N/m^3. The lower plate is fixed, and the upper plate is moving to the right (positive) with constant velocity U= 50 mm/s. Find:

(a) the maximum velocity. (10%)

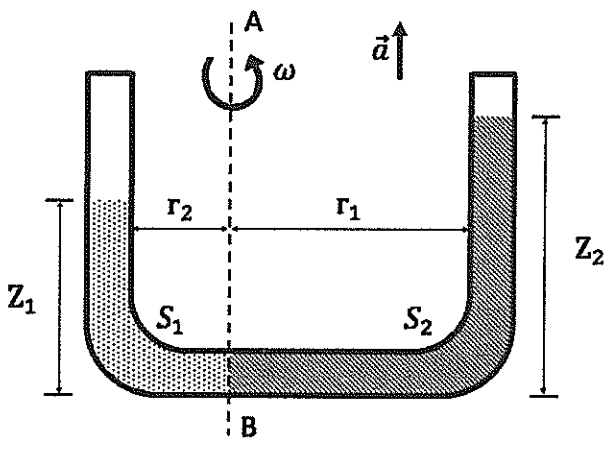
(b) the magnitude and direction of the shear stress on the lower plate. (10%)

4. The U-tube shown is filed with two immiscible fluids, S_1 and S_2, with densities of 1200 kg/m^3 for S_1 and 900 kg/m^3 for S_2. The tube is rotated about vertical axis AB with the angular speed $\omega = 8$ rad/s. For the dimensions shown, if Z_1=20cm, find:

(a) Z_2 (10%)

(b) Z_2, when the U-tube is moving upward with an acceleration of $\vec{a} = 1\ \mathrm{m/s^2}$ (5%)

(c) Z_2, when the U-tube is moved to space (zero gravity) (5%)



5. $F(z) = \Phi(x,y) + i\Psi(x,y)$ is the complex potential which can be used to model the flow in hydrodynamics. $\Phi(x,y)$ is the velocity potential and $\Psi(x,y)$ is the stream function. $F(z) = z^2 = x^2 - y^2 + i(2xy)$, and the complex potential can model "flow around a corner" case. Show the velocity potential function and stream function satisfy Laplace's equation. Plot the stream function on positive (x,y) coordinate. (20%)

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