

1. (15%) Consider current flows with density $\vec{J} = x\vec{a}_x + y\vec{a}_y + 2z\vec{a}_z$ A/m². Find the time rate of decrease of the charge contained within the volume bounded by the cylinders $r=1$ and $r=2$ and the planes $z=0$ and $z=1$.

2. (15%) The electric field of a uniform plane wave is given by

$$\vec{E} = E_0 \cos(\omega t - \beta z + \pi/3)\vec{a}_x - E_0 \cos(\omega t - \beta z + \pi/6)\vec{a}_y$$

Express the wave as a superposition of right- and left-circularly polarized fields.

3. (15%) The magnetic field of a uniform plane wave propagating in the +z-direction in a nonmagnetic material ($\mu = \mu_0$) medium is given by

$$\vec{H} = H_0 e^{-z} \cos(6\pi \times 10^7 t - \sqrt{3}z)\vec{a}_y \text{ A/m}$$

- (a) Find the associated electric field \vec{E} . (5%).
(b) Find the instantaneous power flow across a surface of area 1m² in the $z=0$ plane at $t = 0$ s. (5%).
(c) Find the time average power flow across a surface of area 1m² in the $z=1$ m plane. (5%).
4. (15%) Let us consider the charge distribution given by

$$\rho = \begin{cases} \rho_0 \sin x & \text{for } -\pi < x < \pi \\ 0 & \text{otherwise} \end{cases}$$

where ρ_0 is a constant. Find the potential V . Assume $V = 0$ for $x = 0$.

5. (20%) A parallel-plate transmission line is made up of perfect conductors of width $w=0.1$ m and lying in the planes $x=0$ and $x=0.01$ m. The medium between the conductors is a nonmagnetic perfect dielectric. For a uniform plane wave propagating along the line, the voltage along the line is given by

$$V(z,t) = 10 \cos(3\pi \times 10^8 t - 2\pi z) \text{ V}$$

Neglecting fringing of fields, find

- (a) the electric field intensity $E_x(z,t)$ of the wave. (5%)
(b) the magnetic field intensity $H_y(z,t)$ of the wave. (5%)
(c) the current $I(z,t)$ along the line. (5%)
(d) the power flow $P(z,t)$ down the line. (5%)

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6. (20%) Region 1 ($z < 0$) is free space, whereas region 2 ($z > 0$) is characterized by $\sigma = 10^{-4}$ S/m, $\epsilon = 5\epsilon_0$, $\mu = \mu_0$. For a uniform plane wave having the electric field

$$\vec{E}_i = E_0 \cos(3\pi \times 10^5 t - 10^{-3} \pi z) \vec{a}_x$$

normal incident on the interface $z=0$ from region 1, obtain the expression for the electric and magnetic fields of each of the reflected and the transmitted waves.

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