

1. Find a differentiable function $y = f(x)$ satisfying the Clairaut differential equation $y = xy' - \frac{1}{4}y'^2$ and the boundary value conditions $f(-1) = \frac{3}{4} = f(1)$. (20%)
2. Solve the initial value problem $y' = x^2 + y^2, y(0)=0$ by Picard iteration method:
 $y_1(x) \equiv 0, y_2(x) =?, y_3(x) =?$ (20%)
3. $y = f(x)$ satisfies $f'' + f' = 0$, $z = g(x)$ satisfies $g'' + \frac{x}{x+1}g = 0$ ($x > 0$). Does y oscillate faster than z or z oscillate faster than y ? In the former case you need to show in between my two consecutive zeros of $g(x) = 0$, there exists at least one zero of $f(x) = 0$. (20%)
4. Snow began to fall during the morning of February 2 and continued steadily into the afternoon. At noon a snowplow began removing snow from a road at a constant rate. The plow traveled 6 km from noon to 1 pm but only 3 km from 1 pm to 2 pm. When did the snow begin to fall? (20%)
5. Find a nontrivial solution $y = f(x) \not\equiv 0$ satisfying the Grönwall inequality $|y'| \leq |y|$ with the initial value condition $y(1) = 0$. (20%)

試題隨卷繳回