1. A positronium is a hydrogen-like bound state formed by an electron and its anti-particle, positron. Use Bohr's model to deduce its energy levels. Is the ground state energy identical to that of a hydrogen atom (10%)? The positronium is short-lived. The electron and positron can annihilate each other and release gamma rays. Argue that it cannot emit a single high energy photon (5%). If an observer is traveling at 0.6 c with respect to the positronium and is facing one of the two photons, what will be the photon energy measured by the observer (10%)?

   Some constants: $e = 1.6 \times 10^{-19}$ C, $h = 6.6 \times 10^{-34}$J·s, $c = 3.0 \times 10^{8}$ m/s, $m_e = 0.51$ MeV/c^2, $m_p = 940$ MeV/c^2.

2. There were many famous experiments which played important roles in the development of the quantum mechanics. Name three of them and specifically write down the related contributions to the quantum concept for each experiment (12%). Pick the one you know best to draw the schematic diagram of the experimental setup. You need to briefly describe the experimental procedure (13%).

3. It is known that if $\Psi(\vec{x}, t)$ is a solution of the 3-dimensional single-particle Schrodinger equation, then the following equation holds:

$$\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot \vec{J} = 0,$$

   where $\rho(\vec{x}, t) \equiv \Psi^*(\vec{x}, t)\Psi(\vec{x}, t)$.

   (a) Give the explicit expression for $\vec{J}$. (10%)
   (b) What is the physical meaning of $\vec{J}$? (5%)

4. Consider a particle in a 1-dimensional box. The particle can move freely along the x-axis anywhere between $x = -a/2$ and $x = a/2$, but is forbidden to be found outside this region. Evaluate the expectation value of p^2 when the particle is in the ground state, where p is the momentum operator. (15%)

5. It is known that the $2P_{1/2}$ and $2P_{3/2}$ states of the hydrogen atom have different energies. The spin-orbit interaction is one of the factors that contribute to this difference in energies.

   (a) Write down the complete form of the spin-orbit interaction Hamiltonian. (10%)
   (b) Which state, $2P_{1/2}$ or $2P_{3/2}$, will have higher energy if only the spin-orbit interaction is taken into account? Why? (10%)

試題隨卷繳回