1. (10%) Find the Fourier series of the following function $f(x)$
   $f(x)=x+\pi$ if $-\pi<x<\pi$ and $f(x+2\pi)=f(x)$

2. (a) (4%) Find the Laplace transform for $y(t)=t^2\sin 3t$
   (b) (4%) Find the inverse Laplace transform for $\frac{6s+7}{2s^2+4s+10}$

3. (7%) Using the Laplace transform to solve the following differential equation.
$$y''+3y'+2y=\begin{cases} 4t, & \text{if } 0<t<1 \\ 8, & \text{if } t>1 \end{cases}, \quad y(0)=y'(0)=0$$

4. (10%) Find the solution for the following differential equation by Frobenius method. Identify the series as expansions of known functions.
$$xy''+(2x+1)y'+(x+1)y=0$$

5. (15%) Please use separation of variables solving the following partial differential equation
$$\frac{\partial u(x,t)}{\partial t}=c\frac{\partial^2 u(x,t)}{\partial x^2} \quad \text{for } 0\le x\le L \quad \text{and} \quad t\ge 0$$
   Boundary conditions: $u(0,t)=u(L,t)=0$, for all $t\ge 0$
   Initial condition: $u(x,t=0)=\begin{cases} x, & \text{if } 0<x<L/2 \\ L-x, & \text{if } L/2<x<L \end{cases}$

6. Find the general solutions of the following ordinary differential equations:
   (a) $x^2\frac{d^2y}{dx^2}+3x\frac{dy}{dx}+2y=0$ (8%)
   (b) $D\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dC}{dr}\right)-kC=0$, where D and k are constants (6%)

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7. Consider the following matrix B:

$$B = \begin{bmatrix} 4 & 2 \\ 2 & 1 \end{bmatrix}$$

(4%) (a) Please determine the eigenvalues and eigenvectors of the matrix B

(6%) (b) If the matrix B is similar to D, a diagonal matrix with eigenvalues as the diagonal components), please determine the transition matrix P and its inverse P^{-1} ?

(4%) (c) Please apply the results from (a) and (b) to find B^3

(6%) (d) Applying the results from (a) and (b) for solving the following system of ODEs:

$$\frac{dx_1}{dt} = 4x_1 + 2x_2 + 3e^t$$

$$\frac{dx_2}{dt} = 2x_1 + x_2 + e^t$$

8. Consider a vector field:

$$\mathbf{G}(x, y) = \mathrm{P}\mathbf{i} + \mathrm{Q}\mathbf{j} = (xe^{x^2+y^2} + 2xy)\mathbf{i} + (ye^{x^2+y^2} + x^2)\mathbf{j}$$

(3%) (a) Determine the divergence of **G**, $\nabla \bullet \mathbf{G}$

(4%) (b) Show that $\mathbf{G} = \nabla f$ for some f; please find such an f.

(3%) (c) Use (a) to determine the line integral of **G** around the edge of the triangle with vertices (0; 0); (0; 1); (1; 0)

(3%) (d) State the "Green's theorem"

(3%) (e) Apply the "Green's theorem" for the triangle defined in (b) and verify it for the vector field **G** mentioned above

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