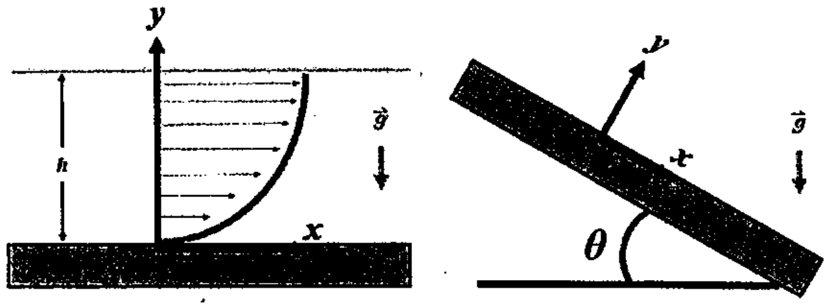
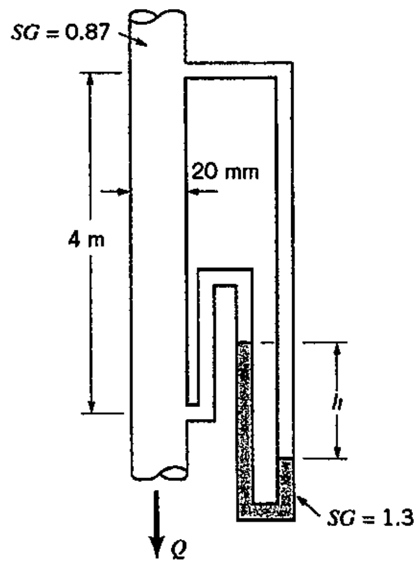
1. **(20%)** As shown in the following figure, consider a steady, two-dimensional, incompressible, viscous liquid flow over a long plate that is influenced by gravity. To understand the flow motion, you may (a) first write down the governing equations in both x and y axes. What are the appropriate boundary conditions you would impose to solve them? (b) Let the plate to incline at an angle θ as shown in the right side of the figure. Assume the velocity u is independent of x. Write down the governing equations in both x and y axes.(c) Show that $p = p_0 + \rho g(h - y)\cos\theta$, where p_0 is the pressure at the free surface $y = h$. (d) From the x-component, using the appropriate boundary conditions at the plate surface and free surface at $y = h$, show that $u_x = \frac{g}{2\nu} y(2h - y)\sin\theta$. (e) Find the volume flux per unit distance along z.



2. **(15%)** For a two-dimensional straining flow with the stream function $\varphi = \alpha xy$ and a simple shear flow with $\varphi = \frac{1}{2}\beta y^2$, where α and β are constants. (a) Find the velocity equations for these flows. (b) At $t = 0$, dye is introduced to mark a curve: $\left(\frac{x}{R}\right)^2 + \left(\frac{y}{R}\right)^2 = 1$. Find the equation(s) to describe the material fluid curve for $t > 0$; interpret how you would derive it (them). Sketch how the curve evolves with time. (c) Does the area inside the curve vary in time? Why? (d) Which of the two flows stretches the curve faster at long times? Please interpret or derive the details whenever you need them.

3. **(15%)** For a laminar viscous flow in a *horizontal* pipe, the volume flowrate can be expressed as $Q = \frac{\pi D^4 \Delta p}{128\mu l}$, where Δp is the pressure drop, μ the viscosity and l the pipe length. Oil of $SG = 0.87$ and a kinematic viscosity $\nu = 2.2 \times 10^{-4}$ m^2/s flows through the vertical pipe shown in the figure at a rate of 4×10^{-4} m^3/s. Determine the manometer reading h. What about the reading h if the flow is up rather than down the pipe.



4. **(24%)** A three-dimensional steady velocity field is given by $u = x^2 + 1$, $v = y$, and $w = x$, where *u, v,* and *w* are the three velocity components in the *x*, *y*, and *z* directions respectively and have a unit of meter per second.

(a) Find the volume change rate per unit volume of a fluid element located at $(x, y, z) = (0,1,1)$ at time *t*=1sec.

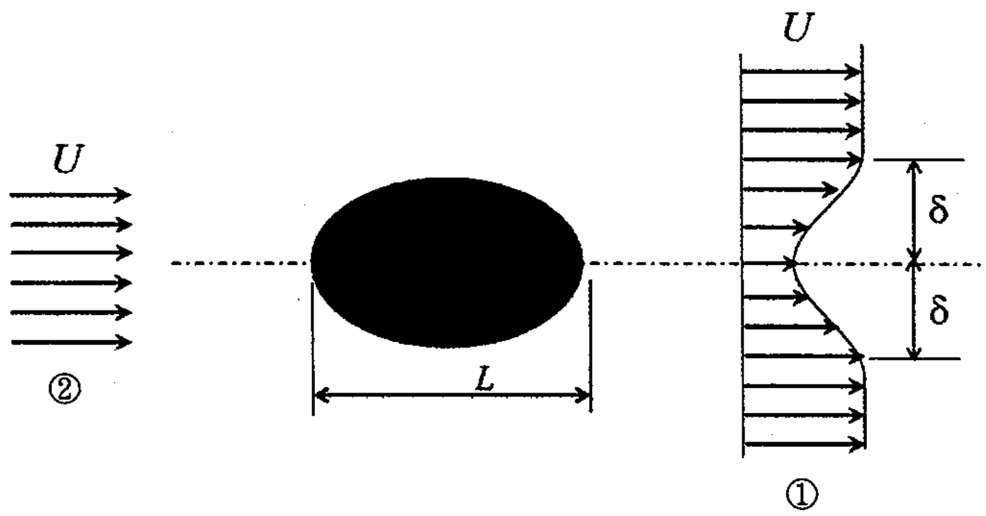
(b) Find the instantaneous acceleration along the streamline direction of the particle located at $(x, y, z) = (0,1,1)$ at time *t*=1sec.

(c) Find the equation of the pathline of the particle which passed through $(x, y, z) = (0,1,1)$ at time *t*=1sec.

5. **(26%)** A uniform stream (density ρ) flows past an immersed elliptical cylinder, creating a broad low-velocity wake downstream as shown in the figure below. Pressures p_1 and p_2 are approximately equal. Assume the flow is two-dimensional, steady, and incompressible.

(a) Based on the principles of mass conservation and momentum conservation, derive a formula for the drag suffered by the cylinder in terms of ρ, *U*, *u*(*y*), and maybe *L*. Describe clearly the control volume you select for your analysis.

(b) Assume the velocity profile in the wake can be idealized as $u = U_0 - (U - U_0)\cos(2\pi y/L)$ for $-L/2 \le y \le L/2$ and $u = U$ otherwise. Find the drag coefficient.



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