1. Given that the Gamma function has the following properties:

$\Gamma(\alpha)\Gamma(1-\alpha)=\frac{\pi}{\sin\alpha\pi}$, $0<\alpha<1$, and $\Gamma\left(n+\frac{1}{3}\right)=\Gamma\left(\frac{1}{3}\right)\frac{(3n-2)!}{3^n}$

Solve the integral equation $3\int_0^t \frac{x(\tau)}{\sqrt[3]{t-\tau}}d\tau=2t$ and express its solution in terms of π and t. (10%)

2. Solve the differential equation $\frac{d^2y}{dx^2}=\sec^2 y\tan y$, with the initial conditions $y(-1)=0$ and $y'(-1)=1$. (15%)

3. Solve the following nonhomogeneous system of linear differential equations:

$$\frac{dx}{dt}=-3x+y+3t$$

$$\frac{dy}{dt}=2x-4y+e^{-t}$$

(15%)

4. Solve the partial differential equation $\frac{\partial u}{\partial t}=k^2\frac{\partial^2 u}{\partial x^2}+F(x)$ for $0\le x\le L$ with $F(x)=x(x-L)$, boundary conditions $u(0,t)=u(L,t)=0$ and the initial condition $u(x,0)=x$. Solution is sought in the form of $u(x,t)=\sum_{n=1}^{\infty}T_n(t)\phi_n(x)$ where $\phi_n(x)$ is the eigenfunction of the problem according to the Sturm-Liouville theorem:

Find $\phi_n(x)$ and the eigenfunction expansion of $F(x)$ and the initial condition (12%)

Find an equation for $T_n(t)$ and solve for the final $u(x,t)$. (6%)

5. Show that the Fourier transformation of $e^{-\alpha t^2}$ is $\sqrt{\frac{\pi}{\alpha}}e^{-\frac{\omega^2}{4\alpha}}$ with constant α. (4%) Solve $\frac{\partial u}{\partial t}=k^2\frac{\partial^2 u}{\partial x^2}$ for $-\infty<x<\infty$ subjected to $u(x,0)=x^2$

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(8%)

6. The velocity field of a fluid is given by $\mathbf{F}(x,y,z) = y\mathbf{i} - x\mathbf{j} + 4\mathbf{k}$. The surface S is that part of the sphere $x^2+y^2+z^2=9$ that is above the region D in the xy plane enclosed by the circle $x^2+y^2=4$
(15%)

(a) Determine the unit outward normal vector of the surface S.

(b) Determine the area of the surface S.

(c) Determine the flux of $\mathbf{F}$ across S.

7. Let $F(z) = \dfrac{1}{(z^2-4)(z-3)^2}$, where z is a complex variable.

(15%)

(a) Identify poles and the order of poles of $F(z)$.

(b) Determine the residue of each pole.

(c) Determine the inverse Laplace transform of $F(z)$. Please draw the integral contour for calculating the inverse Laplace transform.

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