

1. (10 points) A random sample of 20 students yielded a mean of $\bar{x} = 72$ and a variance of $s^2 = 16$ for scores on a college placement test in mathematics. Assuming the scores to be normally distributed, construct a 98% confidence interval for σ^2 .

2. (10 points) Show that $-1 \leq \rho \leq 1$ where ρ is the coefficient of correlation.

3. (20 points) In an accelerated life test, the principal material is aged by exposing it to 70°C for 168 hours. The tensile strength of the specimens of this material is measured before and after the aging process. The following data (in psi) are recorded:

Specimen	1	2	3	4	5	6	7	8	9	10
Original	215	226	226	219	222	231	234	219	209	216
Aged	203	216	217	211	215	218	224	210	201	207

Find a 99% confidence interval (CI) on the mean difference in tensile strength.

- Assume the samples are paired.
- Assume the samples are not paired.
- The pairing of samples usually leads to smaller a CI. Explain why.

4. (20 points) Five delaminated beams made from composite laminates were subjected to loads, and the resulting frequencies were as follows (in Hz): 230.66, 233.05, 232.58, 229.48, 232.58. Is there evidence to support that mean natural frequency is 235Hz?

- Use the approach of hypothesis testing with $\alpha=0.10$.
- Use P-value approach.

5. (20 points) Let X represent the number that occurs when a green die (骰子) is tossed and Y the number that occurs when a red die is tossed. Find

- $E(X+Y)$
- $E(X-Y)$
- $E(XY)$
- $\text{Var}(2X-Y)$
- $\text{Var}(X+3Y-5)$

6. (20 points) The traffic on the one-way main street may be described by a Poisson process with an average arrival rate of 10 cars per minute. A driver on the side street is waiting to cross the main street. He will cross as soon as he finds a gap of 15 sec.

- Determine the probability that a gap will be longer than 15 sec.
- What is the probability that the driver will cross at the fourth gap?
- Determine the mean number of gaps he has to wait until crossing the main street.
- What is the probability that he will cross within the first four gaps?

