

Prob 1.(30%) For the system of equations

$$X_1 + X_2 + X_3 = 0$$

$$X_1 + 2X_2 = 1$$

$$X_1 + 2X_3 = 0$$

$$X_4 = -1$$

Write the above system as $AX=B$ where A , X and B are matrixes and $X^T=(X_1, X_2, X_3, X_4)$.

Answer the following with detailed calculating steps. (No points for answer only)

- (5%) Solve X with $(A | B)$ augmented matrix and use elementary row operation only.
- (6%) Solve A^{-1} .
- (16%) Find the eigenvalue and eigenvectors for A .
- (3%) Express $A^{500} = YDY^{-1}$ and what is matrix Y and D

Prob. 2 (20%) Evaluate $\iint_S \nabla \times \vec{F} \cdot \vec{n} dA$ **with** $\vec{F} = (x, yz, xz)$.

Surface S is defined as $x^2 + y^2 + z^2 = 4, \quad z \geq -1$.

Prob. 3. (15%) Solve $x^2 \frac{d^2 y}{dx^2} - 2x \frac{dy}{dx} + 2y = x$.

- (7%) Solve the general homogeneous solution
- (6%) Solve the non-homogeneous solution
- (2%) Solve y with the initial condition $y(1) = 1, \frac{dy}{dx}(1) = -1$

Prob 4. (15%) $u_{xx} + \lambda u = 0, \quad 0 \leq x \leq 2$ **with** $u_x = 0, x=2, \quad u_x = 0 \quad x=0$

- (13%) Solve all the eigenvalues and eigenfunctions
- (2%) Write down the orthogonal relationship and orthonormal set for eigenfunctions found in (a)

Prob 5. (20%) Solve $u_{xx} - 4u_{yy} = 0 \quad -\infty < x < \infty, \quad 0 \leq y < \infty$

(a)(15%) With the initial condition at $y=0$ $\begin{cases} u = 1, & -2 \leq x < 0 \\ u = -1, & 0 \leq x \leq 2 \\ u = 0, & \text{otherwise} \end{cases}$

(b)(5%) Draw a graph in (x,y,u) coordinate to show the solution