1. (7%) (8%) Let $X_{11}, \ldots, X_{1n_1}$ and $X_{21}, \ldots, X_{2n_2}$ be two independent random samples from populations 1 and 2 with the corresponding distributions $F_1(x)$ and $F_2(x)$. Derive the mean and variance of $T_1 = \sum_{j=1}^{n_1} R(X_{1j})$ under $F_1(x) = F_2(x)$, where $R(X_{1j})$ is the rank of X_{1j} among $\{X_{ij} : i = 1, 2; j = 1, \ldots, n_i\}$.

2. (20%) Let X_1, X_2 and X_3 be a random sample from a $Poisson(\lambda)$. Moreover, let $Y_1 = X_1 + X_3$, $Y_2 = X_2 + X_3$, and $Z_i = I(Y_i > 0)$, $i = 1, 2$. Compute the correlation of Z_1 and Z_2.

3. (15%) Let $X_1, \ldots, X_n$ be a random sample from a $Bernoulli(\pi)$, $0 < \pi < 1$. Find the smallest sample size to achieve $P(|\widehat{\pi}_n - \pi| \le e) \approx 1 - \alpha$, where $\widehat{\pi}_n$ is the sample mean.

4. (20%) Let $X_1, \ldots, X_n$ be a random sample from a density function

$$f(x|\theta) = \theta e^{-\theta x} I_{(0,\infty)}(x), \quad 0 < \theta < \infty.$$

Derive the sampling distribution of the uniformly minimum variance unbiased estimator of $1/\theta$.

5. Let $X_1, \ldots, X_n$ be a random sample from a normal distribution with mean μ and variance σ^2, where σ^2 is an unknown constant. Consider the null hypothesis $H_0 : \mu \ge \mu_0$ versus the alternative hypothesis $H_A : \mu < \mu_0$.

(5a) (8%) Derive the likelihood ratio test with size α, $0 < \alpha < 1$.

(5b) (7%) Express the corresponding p-value of the likelihood ratio test based on observed values $x_1, \ldots, x_n$ of a random sample.

(5c) (7%) Compute the power at μ_1 with $\mu_1 < \mu_0$.

(5d) (8%) Construct the uniformly most accurate $(1 - \alpha)$ confidence interval of μ.

試題隨卷繳回