# 國立臺灣大學 105 學年度碩士班招生考試試題

科目：微積分(A)

節次： 4



(1) (10%) Find the limit $\lim_{x\to 0} x(\frac{1}{x^3} - \frac{1}{\sin^3 x})$.

(2) (10%) Evaluate the integral $\int \frac{dx}{\sin x + 8\cos x + 4}$.

(3) (10%) Evaluate the integral

$$\int_{-2}^{2}\int_{0}^{\sqrt{4-y^2}}\int_{-\sqrt{4-x^2-y^2}}^{\sqrt{4-x^2-y^2}} y^2\sqrt{x^2+y^2+z^2}\,dzdxdy.$$

(4) (10%) Find all normal lines of $y = x^3 + 2x^2$ which pass through the point $(-2, 0)$.

(5) (10%) Find the relative extreme values and the absolute extreme values of

$$f(x) = (1 + x + \frac{x^2}{2!} + \cdots + \frac{x^n}{n!})e^{-x}, x \in \mathbb{R},$$

where $n$ is a positive integer.

(6) (10%) Find the sum of the series $\frac{x}{1\cdot 2} + \frac{x^2}{2\cdot 3} + \frac{x^3}{3\cdot 4} + \cdots + \frac{x^n}{n(n+1)} + \cdots$.

(7) (10%) The plane $x + y + z = a$ cuts the unit sphere $x^2 + y^2 + z^2 = 1$ into two pieces $S_1$ and $S_2$, where $0 \le a \le \sqrt{3}$. Compute $|\text{area}(S_1) - \text{area}(S_2)|$.

(8) (10%) Find the area of the region enclosed by the curve $x^3 + y^3 = 3xy$.

(9) (10%) An 80 kg man carries a 5 kg can of paint up a helical staircase that encircles a silo with a radius 10 m. Suppose there is a hole in the can of paint and 2 kg of paint leaks steadily out of the can during the man' ascent. If the silo is 40 m high and the man makes exactly four complete revolutions climbing to the top, how much work is done by the man against gravity?

(10) (10%) A particle travelling in a straight line with constant velocity $-\mathbf{i} - 3\mathbf{j} + 5\mathbf{k}$ passes through the point $(\frac{5}{2}, 4, -\frac{7}{4})$ and hit the surface $z = x^2 + y^2$. The particle ricochets off the surface, the angle of reflection being equal to the angle of incidence. Assuming no loss of speed, what is the velocity of the particle after the ricochet?

試題隨卷繳回