*各題答案應作答於答案卡，否則不予計分。
*每題有一個或一個以上正確選項，完整答對(無任何選項答錯)，該題得滿分。
*每題未作答或答錯(應選而未選或不應選而選)，該題以 0 分計。

1. (10%) A semi-infinite plate coincides with the region defined by $0 \le x \le \pi, y \ge 0$. The left end is at temperature, $u(0,y) = e^{-y}$, and the right end is held at temperature zero. The bottom of the plate is insulated. Please solve the boundary-value problem for the stead-state temperature $u(x,y)$.

   $u(x,y) = \frac{2}{\pi}\int_0^{\infty} \frac{\sinh[s(\alpha)]}{[t(\alpha)]\sinh[r(\alpha)]}\cos(\alpha y)\,d\alpha.$

   (A) $r(\alpha) = \alpha x$
   (B) $s(\alpha) = \alpha(\pi - y)$
   (C) $s(\alpha) = \alpha(\pi - x)$
   (D) $t(\alpha) = 1 + \alpha^2$
   (E) $t(\alpha) = 1 + \alpha x$

2. (10%)Use the result $\int_{-\infty}^{+\infty} \exp(-x^2)dx = \sqrt{\pi}$ to find the Fourier integral transform $F(\alpha) = \int_{-\infty}^{+\infty} \exp[-x^2/(4p^2)]\exp(i\alpha x)dx = s(p)\sqrt{\pi}\exp[-t(p)\alpha^2]$
   (A) $s(p) = -2p$
   (B) $s(p) = p^2$
   (C) $s(p) = 2p$
   (D) $t(p) = 4p^2$
   (E) $t(p) = p^2$

3. (5%) The vertical displacement $u(x,t)$ of an infinitely long string is determined from the initial-value problem $a^2\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2}$. The string is released from rest from the initial displacement $f(x) = \exp(-x^2)$.

   $$u(x,t) = \exp[-x^2 + s(t)]\cosh[r(t)x]$$

   (A) $s(t) = -a^2t^2$
   (B) $s(t) = -2at$
   (C) $r(t) = -2at$
   (D) $r(t) = at$
   (E) $r(t) = 2at$

4. (10%) Consider a dynamic system with a time-varying state $x(t)$ which satisfies the following differential equation:

   $$(t^2 - 3)x'' + 4tx' + 2x = 0, \quad t \ge 0.$$

   Under the conditions $x(0) \in (1,3)$ and $x'(0) \in (-1,1)$, find **all possible values** of $x(3)$ from below.
   (A) -4 (B) -2 (C) 0 (D) 1/2 (E) 1
   Hint: Use power series to solve $x(t)$ and simplify your solution.

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5. (10%) Solve the following initial value problem: $y(0) = 1, y'(0) = 3$, and

$$y'' - y = g(x) = \begin{cases} 0 & x < 0 \\ x^3 & x \geq 0 \end{cases}.$$

Choose from below the value of $(y(-1), y(1))$.

(A) $(5e - 4e^{-1} - 7, 5e^{-1} - 4e + 7)$

(B) $(5e - 4e^{-1} - 7, 2e^{-1} - e)$

(C) $(2e - e^{-1}, 5e^{-1} - 4e + 7)$

(D) $(2e - e^{-1}, 2e^{-1} - e)$

(E) None of the above.

6. (5%) A function $y(x)$ satisfies $(y')^2 = 9x^4 y$, $y(0) = 0$. Find all possible values of $y'(-2)$ from below.

(A) -16 (B) 0 (C) 24 (D) -32 (E) -48

7. (5%) Let X be a random integer from the set $\{1, 2, 3, \ldots, N\}$. Which of the statements below is(are) TRUE?

(A) $E(\mathrm{X}) = N/2$

(B) $E(\mathrm{X}) = (N-1)/2$

(C) $Var(\mathrm{X}) = N^2/4$

(D) $Var(\mathrm{X}) = (N^2 - 1)/4$

(E) None of the above

8. (5%) A joint probability density function of X and Y is given by

$$f(x,y) = \begin{cases} \frac{1}{49} e^{-y/7} & \text{if } 0 \leq x \leq y < \infty \\ 0 & \text{elsewhere} \end{cases}$$

Which of the following statements is(are) TRUE?

(A) $E(\mathrm{X}) = 7$

(B) $E(\mathrm{Y}) = 17$

(C) $E(\mathrm{XY}) = 137$

(D) $Cov(X, Y) = 49$

(E) None of the above

9. (10%) The joint probability density function of random variable X and Y is given by

$$f(x,y) = \begin{cases} \lambda x^2 y & 0 \leq y \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

Find λ and the respective marginal probability density functions of X and Y ($f_X(x)$ and $f_Y(y)$).

(A) $\lambda = 10$

(B) $f_X(x) = 5x(1 - x^3)$, $0 \leq x \leq 1$

(C) $f_X(x) = \frac{10}{3} x^4$, $0 \leq x \leq 1$

(D) $f_Y(y) = 5y^4$, $0 \leq y \leq 1$

(E) $f_Y(y) = \frac{10}{3} y(1 - y^3)$, $0 \leq y \leq 1$

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10. (5%) Find the expected value μ and the variance σ^2 of a random variable with the probability function.

$$f(x) = \sqrt{\frac{2}{\pi}} e^{-2x^2+4x-2}$$

(A) $\mu = \pi/2$ (B) $\mu = 1/2$ (C) $\sigma^2 = 1/2$ (D) $\sigma^2 = 1/4$
(E) None of the above

11. (5%) Let X and Y be continuous random variables with joint probability density function

$$f(x,y) = \begin{cases} \frac{3}{2}(x^2+y^2) & if\ 0<x<1, 0<y<1 \\ 0 & otherwise \end{cases}$$

Find the conditional probability function $f_{Y|X}(y|x) = \frac{3x^2+s(x,y)}{t(x,y)}$.

(A) $s(x,y) = 3y^2+1$
(B) $s(x,y) = 3x^2+1$
(C) $t(x,y) = 2x^2+2y^2$
(D) $t(x,y) = 3y^2$
(E) None of the above

12. (10%) Let Z be a standard normal random variable and X be an exponential random variable with parameter λ. Find the moment-generating function of Z and X.

(A) $M_Z(t) = e^{t^2}$
(B) $M_Z(t) = e^{t^2/2}$
(C) $M_X(t) = \lambda/(\lambda - t)$
(D) $M_X(t) = \lambda/(\lambda - t/2)$
(E) None of the above
Hint: The probability density function of X is given by $f(x) = \lambda e^{-\lambda x}$

13. (10%) Given a density function

$$f(x) = \begin{cases} \lambda x e^{-x} & x > 0 \\ 0 & otherwise \end{cases}$$

Find λ and the cumulative distribution function $F(x)$.

(A) λ=1/4
(B) λ=1/2
(C) $F(x) = 0$ if $x < 0$
(D) $F(x) = -(x/2+1)e^{-x/2}+1$ if $x \geq 0$
(E) $F(x) = -(x+1)e^{-x}+1$ if $x \geq 0$

試題隨卷繳回