

※ 注意：請用 2B 鉛筆作答於答案卡，並先詳閱答案卡上之「畫記說明」。

Multiple Choice: There is only one correct answer to each of the problems below.

- (5 分) The distance between the parallel planes $2x + y + 2z = 1$ and $2x + y + 2z = 4$ is
(A) 3 (B) $\frac{3}{\sqrt{3}}$ (C) $\frac{3}{\sqrt{5}}$ (D) 1 (E) none of the above.
- (5 分) The matrix $\begin{bmatrix} a^2 & ab & ac \\ ab & b^2 & bc \\ ac & bc & c^2 \end{bmatrix}$, where $a, b, c \in \mathbb{R} - \{0\}$, has
(A) three real, non-zero eigenvalues (B) complex eigenvalues (C) two non-zero eigenvalues (D) one non-zero eigenvalue (E) none of the above.
- (5 分) If A and B are symmetric, which of the following is not necessarily symmetric?
(A) $A+B$ (B) $A+2B$ (C) BA (D) A^2 (E) none of the above.
- (5 分) If $A^2 = A$, then the only possible eigenvalues of A are
(A) ± 1 (B) $\pm i$ (C) 0, 1 (D) 0, -1 (E) none of the above.
- (5 分) Which of the following matrices is similar to $A = \begin{bmatrix} 1 & 3 \\ 2 & 2 \end{bmatrix}$?
(A) $\begin{bmatrix} 4 & 2 \\ 2 & 1 \end{bmatrix}$ (B) $\begin{bmatrix} 4 & 0 \\ 0 & 1 \end{bmatrix}$ (C) $\begin{bmatrix} 4 & 0 \\ 0 & -1 \end{bmatrix}$ (D) $\begin{bmatrix} 1 & 1 \\ 0 & 4 \end{bmatrix}$ (E) none of the above.
- (5 分) The only value of k for which $A = \begin{bmatrix} 1 & 1 \\ 0 & k \end{bmatrix}$ is not diagonalizable is
(A) $k=1$ (B) $k=-1$ (C) $k=0$ (D) $k=2$ (E) none of the above.

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7. (5 分) Let $A = \begin{bmatrix} -3 & 1 & -1 \\ -7 & 5 & -1 \\ -6 & 6 & -2 \end{bmatrix}$. Then the eigenvalues of A^{-1} are
(A) $\frac{1}{16}, \frac{1}{4}, \frac{1}{4}$ (B) $\frac{1}{4}, \frac{1}{2}, \frac{1}{2}$ (C) $\frac{1}{4}, -\frac{1}{2}, -\frac{1}{2}$ (D) $-\frac{1}{4}, \frac{1}{2}, \frac{1}{2}$ (E) none of the above.

8. (5 分) Let $S: \mathbb{R}^2 \rightarrow M_{22}$, $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ and $U: M_{22} \rightarrow \mathbb{R}^2$ be linear transformations. Which of the following compositions is not defined?
(A) $S \circ T \circ U$ (B) $T \circ U \circ S$ (C) $S \circ T \circ T$ (D) $S \circ S \circ U$ (E) none of the above.

9. (5 分) Let $T: P_2 \rightarrow \mathbb{R}^2$ be defined by $T(p(x)) = \begin{bmatrix} p(0) \\ p(1) \end{bmatrix}$, where P_2 is a set consisting of the zero polynomial and all polynomials of degree less than or equal to 2. Then $\text{nullity}(T) =$
(A) $x(x-1)$ (B) x (C) 1 (D) 0 (E) none of the above.

10. (5 分) Consider the space P_2 with inner product
$$\langle p, q \rangle = p(0)q(0) + p\left(\frac{1}{2}\right)q\left(\frac{1}{2}\right) + p(1)q(1).$$

A vector orthogonal to $p = 4x^2 - 1$ is

- (A) $x^2 - x$ (B) $x^2 - 1$ (C) $4x^2 - 4x + 1$ (D) $2x + 3$ (E) none of the above.

11. (10 分) Consider a random X with a probability density function as

$$f_x(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{(x-3)^2}{2}}$$

Let $Y=2X-6$. Please find the (mean, variance) of random variable Y .

- (A) (0,4) (B) (3,4) (C) (3, 8) (D) (6, 8) (E) none of the above.

12. (10 分) 10 guests join a party in a rainy night. Each of them brings an umbrella. At the end of the party, each guest randomly picks an umbrella. Let X be the number of guests that get their own umbrellas back.
(A) $E[X]=1/2$ (B) $E[X]=1$ (C) $E[X]=2$ (D) $E[X]=5$ (E) none of the above.

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13. (10分) Let X be the number of earthquakes in Taipei and X follows a distribution

$$f_x(x) = 2^x \frac{e^{-2}}{x!} \text{ for } x \geq 0.$$

Whenever an earthquake occurs, the probability that a tsunami also occurs is 0.5.

What is the distribution of Y , the number of tsunamis in Taipei?

- (A) $f_Y(y) = 2^y \frac{e^{-1}}{y!}$ (B) $f_Y(y) = \frac{e^{-2}}{y!}$ (C) $f_Y(y) = 0.5^y \frac{e^{-0.5}}{y!}$
(D) $f_Y(y) = 0.5^y \frac{e^{-1}}{2y!}$ (E) none of the above.

14. (10分) Based on a sample of 1220 potential voters, the percentage of peoples supporting Candidate Tsai is 58% with an accuracy of plus or minus 4 percentage points. What is the confidence probability of this poll?

(A) 0.72 (B) 0.77 (C) 0.82 (D) 0.87 (E) none of the above.

15. (10分) A prisoner is kept in Room A. Room A is directly connected to Room B and Room C, respectively, while both of these two rooms (B and C) are directly connected to Room D, where the police stays. Note that Rooms A and D are not directly connected. Assume that a friend of the prisoner has opened up an escape tunnel from Room C. The prisoner decides to escape at night. Since there is no light, the prisoner would go to either room with a probability of 1/2 when he decides to enter the next room. If the prisoner goes to Room D, he will be executed. If he reaches Room C, he will be free. What is the probability that he can escape successfully?

(A) 1/2 (B) 2/3 (C) 3/4 (D) 4/5 (E) none of the above.

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