1. (10 points) Compute

$$\sum_{i=2}^{n}\sum_{j=2}^{i} C_i^n C_{i-j}^i C_2^j$$

where C_k^n is the coefficient of the x^k term in the expansion of $(1+x)^n$

2. (15 points) Solve the following recurrence:

$$\begin{aligned} a_0 &= 10, \\ a_1 &= 16, \\ a_n &= 5a_{n-1} - 6a_{n-2} + 4n, \text{ for all } n \geq 2. \end{aligned}$$

3. (10 points) Find the smallest $x \in \mathbb{N}$ such that

$$\begin{aligned} x &\equiv 1 \pmod{4} \\ x &\equiv 2 \pmod{5} \\ x &\equiv 15 \pmod{19}. \end{aligned}$$

Show your derivation.

4. (35 points) For each of the following statements, determine whether it is true of false. No explanation is needed. You get +5 points for every correct answer and -6 points for every incorrect one. (0 points if you do not answer.)

   (a) $\exists x \forall y P(x,y) \rightarrow \forall y \exists x P(x,y)$.

   (b) $\{\rightarrow, \neg\}$ is a functionally complete set.

   (c) There exists two primes p, q such that $(p-1)^q \equiv 100 \pmod{pq}$.

   (d) If S and T are two countable sets, then $S \times T$ must also be countable.

   (e) There exists a bijection between the set of all integers and the set of all rational numbers.

   (f) If a relation R is transitive, then R^{-1} is also transitive. ($R^{-1} = \{(a,b) | (b,a) \in R\}$.)

   (g) The symmetric closure of a transitive relation must be transitive.

5. (15 points) Let G be a tree with n vertices. Suppose that G has no degree-2 vertices, what is the minimum number of leaves that G must have? Prove your answer.

6. (15 points) Let G be a simple undirected planar graph with 9 vertices. Suppose that every vertex in G have the same degree. Prove or disprove that the complementary graph $\bar{G}$ must have a Hamiltonian cycle. Recall that the definition of the complementary graph is the following:

   The complementary graph $\bar{G}$ of a simple graph G has the same vertices as G. Two vertices are adjacent in $\bar{G}$ if and only if they are not adjacent in G.

試題隨卷繳回