

*各題答案應作答於答案卡，否則不予計分。

*每題有一個或一個以上正確選項，完整答對(無任何選項答錯)，該題得滿分。

*每題未作答或答錯(應選而未選或不應選而選)，該題以 0 分計。

1. (10%) A semi-infinite plate coincides with the region defined by $0 \leq x \leq \pi, y \geq 0$. The left end is at temperature, $u(0, y) = e^{-y}$, and the right end is held at temperature zero. The bottom of the plate is insulated. Please solve the boundary-value problem for the steady-state temperature $u(x, y)$.

$$u(x, y) = \frac{2}{\pi} \int_0^{\infty} \frac{\sinh[s(\alpha)]}{[t(\alpha)\sinh[r(\alpha)]} \cos(\alpha y) d\alpha.$$

- (A) $r(\alpha) = \alpha x$, (B) $s(\alpha) = \alpha(\pi - y)$, (C) $s(\alpha) = \alpha(\pi - x)$, (D) $t(\alpha) = 1 + \alpha^2$,
(E) $t(\alpha) = 1 + \alpha x$

2. (10%) Use the result $\int_{-\infty}^{+\infty} \exp(-x^2) dx = \sqrt{\pi}$ to find the Fourier integral

$$\text{transform } F(\alpha) = \int_{-\infty}^{+\infty} \exp[-x^2/(4p^2)] \exp(i\alpha x) dx = s(p) \sqrt{\pi} \exp[-t(p)\alpha^2]$$

- (A) $s(p) = -2p$, (B) $s(p) = p^2$, (C) $s(p) = 2p$, (D) $t(p) = 4p^2$, (E) $t(p) = p^2$

3. (5%) The vertical displacement $u(x, t)$ of an infinitely long string is determined from the initial-value problem $a^2 \frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2}$. The string is released from rest from the initial displacement $f(x) = \exp(-x^2)$.

$$u(x, t) = \exp[-x^2 + s(t)] \cosh[r(t)x]$$

- (A) $s(t) = -a^2 t^2$, (B) $s(t) = -2at$, (C) $r(t) = -2at$, (D) $r(t) = at$,
(E) $r(t) = 2at$

4. (10%) Consider a dynamic system with a time-varying state $x(t)$ which satisfies the following differential equation:

$$(t^2 - 3)x'' + 4tx' + 2x = 0, \quad t \geq 0.$$

Under the conditions $x(0) \in (1, 3)$ and $x'(0) \in (-1, 1)$, find all possible values of $x(3)$ from below.

- (A) -4 (B) -2 (C) 0 (D) 1/2 (E) 1

Hint: Use power series to solve $x(t)$ and simplify your solution.

見背面

5. (10%) Solve the following initial value problem: $y(0) = 1, y'(0) = 3$, and

$$y'' - y = g(x) = \begin{cases} 0 & x < 0 \\ x^3 & x \geq 0 \end{cases}.$$

Choose from below the value of $(y(-1), y(1))$.

- (A) $(5e - 4e^{-1} - 7, 5e^{-1} - 4e + 7)$
 (B) $(5e - 4e^{-1} - 7, 2e^{-1} - e)$
 (C) $(2e - e^{-1}, 5e^{-1} - 4e + 7)$
 (D) $(2e - e^{-1}, 2e^{-1} - e)$
 (E) None of the above.
6. (5%) A function $y(x)$ satisfies $(y')^2 = 9x^4y$, $y(0) = 0$. Find all possible values of $y'(-2)$ from below.
 (A) -16 (B) 0 (C) 24 (D) -32 (E) -48
7. (10%) Use Gaussian elimination procedures to find the nullity of the matrix below:

$$\begin{bmatrix} 1 & 0 & -2 & -1 & 0 & -1 \\ 2 & -1 & -6 & -2 & 0 & -4 \\ 0 & 1 & 2 & 1 & 1 & 1 \\ -1 & 2 & 6 & 3 & 1 & 2 \end{bmatrix}$$

- (A) 0, (B) 1, (C) 2, (D) 3, (E) 4, (F) 5

8. (15%) Suppose that $T: \mathbb{R}^3 \rightarrow \mathbb{R}^3$ is the reflection of \mathbb{R}^3 about the x - y plane. Which of the following statements are true (multiple choice):

(A) The null space of T is $\left\{ \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\}$,

(B) T is one-to-one,

(C) T is onto,

(D) The standard matrix of T is $\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

接次頁

9. (10%) Given the matrix below

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

which of the following statements is (are) true?

- (A) The columns form an orthonormal basis for the plane \mathbb{R}^3 .
(B) The rows are orthonormal.

- (C) The inverse of this matrix is $\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

- (D) The inverse of this matrix is its transpose.
(E) None of the above.

10. (10%) For the matrix below

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

which of the following statements is (are) true?

- (A) 1 is its eigenvalue.
(B) -1 is its eigenvalue.
(C) It has only real eigenvalue(s).
(D) It has complex eigenvalue(s).
(E) None of the above.

11. (5%) The eigenvectors of matrix A are put into the columns of the eigenvector matrix S . If the eigenvectors of A are linearly independent, then which of the following statements is (are) true?

- (A) S is invertible.
(B) S is diagonalizable.
(C) A is invertible.
(D) A is diagonalizable.
(E) None of the above.

試題隨卷繳回