請於答案卷上作答，於試題卷上作答者，不予計分。

1. Consider the inverted pendulum system shown in **Fig. 1**. Assume that the mass of the inverted pendulum is m and is evenly distributed along the length of the rod. (The center of gravity of the pendulum is located at the center of the rod.) Assuming that θ is small, (1) derive mathematical models for the system in the forms of differential equations, (2) transfer functions, i.e., $X(s)/U(s)$ and $\Theta(s)/U(s)$, and (3) state-space equations. Use the state vector $\mathbf{x} = [\theta, \theta', x, x']^T$. 【計分：20 分】

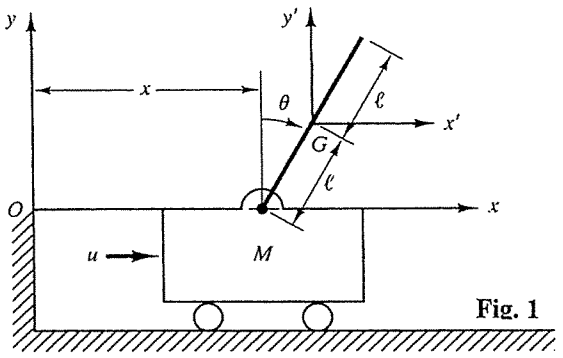

Fig. 1

2. Consider the closed-loop system shown in **Fig. 2**. Determine the range of K for stability. Assume that $K > 0$. 【計分：10 分】

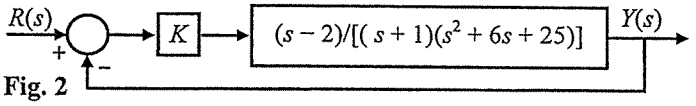

Fig. 2

3. Consider the following state equation and output equation:

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{bmatrix} = \begin{bmatrix}-6 & 1 & 0\\ -11 & 0 & 1\\ -6 & 0 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix} + \begin{bmatrix}2\\ 6\\ 2\end{bmatrix}u;\quad y = \begin{bmatrix}1 & 0 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}$$

Show that the state equation can be transformed into the following form by use of a proper transformation matrix:

$$\begin{bmatrix}\dot{z}_1\\ \dot{z}_2\\ \dot{z}_3\end{bmatrix} = \begin{bmatrix}0 & 0 & -6\\ 1 & 0 & -11\\ 0 & 1 & -6\end{bmatrix}\begin{bmatrix}z_1\\ z_2\\ z_3\end{bmatrix} + \begin{bmatrix}1\\ 0\\ 0\end{bmatrix}u$$

Then obtain the output y in terms of z_1, z_2, and z_3. 【計分：20 分】

4. Given a linear input/output system: $\frac{dx}{dt} = Ax + Bu, y = Cx + Du$,

where $A = \begin{bmatrix}-4 & 2\\ -2 & -1\end{bmatrix}, B = \begin{bmatrix}0\\ 2\end{bmatrix}, C = \begin{bmatrix}0 & 1\end{bmatrix}, D = 0$

Please find the rising time, overshoot and the steady state error of the system output for a unit step input 【計分：15 分】

5. Plot the root locus for the system in the figure below. 【計分：7 分】 Using the root locus, please determine the gain K so that the rising time of the output response of the system is less than 0.1sec. 【計分：8 分】

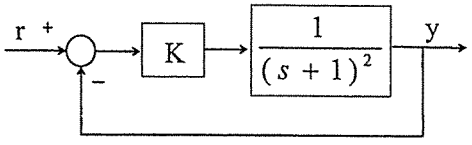

Figure for problem 5.

6. if the open loop transfer function $KG(s) = \frac{1}{s(s+1)^2}$ (1) Please plot the Bode magnitude and phase for the open loop system and find the phase margin and gain margin 【計分：10 分】 (2) Please plot the Nyquist plot of the open loop system and discuss the stability of the close loop system. 【計分：10 分】

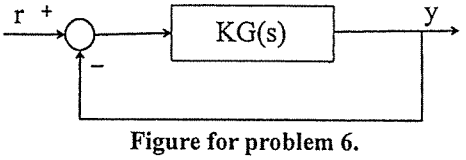

Figure for problem 6.

試題隨卷繳回