

- 1、解釋名詞：
  - (1) 中央極限定理(Central Limit Theorem)。(5分)
  - (2) 動差母函數(Moment Generating Function)。(5分)
- 2、 $X_1, X_2, \dots, X_n$  為來自幾何分配之隨機樣本， $f(x; p) = (1-p)^{x-1}p, x=1, 2, 3, \dots$ 。  
試求  $p$  之最大概似推定量(maximum likelihood estimator)。(10分)
- 3、 $X_1, X_2, \dots, X_n$  為來自  $N(\mu, \sigma^2)$  之隨機樣本， $\bar{X}$  為樣本平均數。  
試求  $\sum_{i=1}^n \left(\frac{X_i - \mu}{\sigma}\right)^2$  之機率分配。(10分)
- 4、Let  $Y$  be  $b(100, p)$ . To test  $H_0: p=0.08$  against  $H_1: p < 0.08$ , We reject  $H_0$  and accept  $H_1$  if and only if  $Y \leq 6$ 。
  - (a) Determine the significance level  $\alpha$  of the test.(5分)
  - (b) Find the probability of the Type II error if in fact,  $p=0.04$ . (5分)
- 5、 $X$  and  $Y$  are independent variables with distribution that are  $N(\mu_X, \sigma_X^2)$  and  $N(\mu_Y, \sigma_Y^2)$ 。 We wish to test  $H_0: \sigma_X^2 = \sigma_Y^2$  against  $H_1: \sigma_X^2 > \sigma_Y^2$ 。  
if  $n=m=31, \bar{x}=8.153, S_x^2=1.410, \bar{y}=5.917, S_y^2=0.4399$ . Please show the test result.  
(10分)
6. An excellent free-throw shooter attempts several free throws until she misses.
  - (a) If  $p=0.9$  is her probability of making a free throw, what is the probability of having the first miss on the 13<sup>th</sup> attempt or later? (6 points)
  - (b) If she continues shooting until she misses three, what is the probability that the third miss occurs on the 30<sup>th</sup> attempt? (6 points)
7. An insurance company sells an automobile policy with a deductible of one unit. Let  $X$  be the amount of the loss having probability mass function (pmf)
 
$$f(x) = \begin{cases} 0.9, & x = 0 \\ \frac{c}{x}, & x = 1, 2, 3, 4, 5, 6, \end{cases} \quad \text{where } c \text{ is a constant.}$$
  - (a) Determine  $c$  (6 points)
  - (b) Calculate the expected value of the amount the insurance company must pay. (6 points)
8. Let  $X$  equal the number of alpha particles emitted by barium-133 per second and counted by a Geiger counter. Assume that  $X$  has a Poisson distribution with  $\lambda = 49$ .  
Use the normal approximation to find  $P(45 \leq X \leq 60)$ . (8 points)

見背面

題號： 327

國立臺灣大學105學年度碩士班招生考試試題

科目：統計學(E)

題號：327

節次： 2

共 4 頁之第 2 頁

9. Let  $X$  and  $Y$  have the joint probability mass function (joint pmf)

$$f(x, y) = \frac{x+y}{21}, \quad x = 1, 2, 3 \quad y = 1, 2$$

(a) Find the marginal pmfs of  $X$  and  $Y$ , that is  $f_X(x)$  and  $f_Y(y)$ . (4 points)

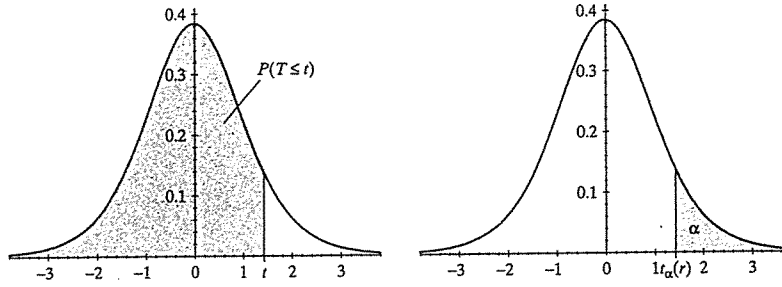
(b) Find  $h(y|x)$ , the conditional pmfs of  $Y$  given  $X=x$ , for  $x=1, 2$  and  $3$ . (4 points)

(c) Find  $P(Y=2|X=1)$  and  $P(Y \leq 2|X=2)$ . (4 points)

(d) Find  $E(Y|X=3)$  and  $\text{Var}(Y|X=3)$ . (6 points)

接次頁

TABLE VI: The t Distribution



$$P(T \leq t) = \int_{-\infty}^t \frac{\Gamma[(r+1)/2]}{\sqrt{\pi r} \Gamma(r/2) (1+w^2/r)^{(r+1)/2}} dw$$

$$P(T \leq -t) = 1 - P(T \leq t)$$

r	P(T ≤ t)						
	t <sub>0.40</sub> (r)	t <sub>0.25</sub> (r)	t <sub>0.10</sub> (r)	t <sub>0.05</sub> (r)	t <sub>0.025</sub> (r)	t <sub>0.01</sub> (r)	t <sub>0.005</sub> (r)
1	0.325	1.000	3.078	6.314	12.706	31.821	63.657
2	0.289	0.816	1.886	2.920	4.303	6.965	9.925
3	0.277	0.765	1.638	2.353	3.182	4.541	5.841
4	0.271	0.741	1.533	2.132	2.776	3.747	4.604
5	0.267	0.727	1.476	2.015	2.571	3.365	4.032
6	0.265	0.718	1.440	1.943	2.447	3.143	3.707
7	0.263	0.711	1.415	1.895	2.365	2.998	3.499
8	0.262	0.706	1.397	1.860	2.306	2.896	3.355
9	0.261	0.703	1.383	1.833	2.262	2.821	3.250
10	0.260	0.700	1.372	1.812	2.228	2.764	3.169
11	0.260	0.697	1.363	1.796	2.201	2.718	3.106
12	0.259	0.695	1.356	1.782	2.179	2.681	3.055
13	0.259	0.694	1.350	1.771	2.160	2.650	3.012
14	0.258	0.692	1.345	1.761	2.145	2.624	2.997
15	0.258	0.691	1.341	1.753	2.131	2.602	2.947
16	0.258	0.690	1.337	1.746	2.120	2.583	2.921
17	0.257	0.689	1.333	1.740	2.110	2.567	2.898
18	0.257	0.688	1.330	1.734	2.101	2.552	2.878
19	0.257	0.688	1.328	1.729	2.093	2.539	2.861
20	0.257	0.687	1.325	1.725	2.086	2.528	2.845
21	0.257	0.686	1.323	1.721	2.080	2.518	2.831
22	0.256	0.686	1.321	1.717	2.074	2.508	2.819
23	0.256	0.685	1.319	1.714	2.069	2.500	2.807
24	0.256	0.685	1.318	1.711	2.064	2.492	2.797
25	0.256	0.684	1.316	1.708	2.060	2.485	2.787
26	0.256	0.684	1.315	1.706	2.056	2.479	2.779
27	0.256	0.684	1.314	1.703	2.052	2.473	2.771
28	0.256	0.683	1.313	1.701	2.048	2.467	2.763
29	0.256	0.683	1.311	1.699	2.045	2.462	2.756
30	0.256	0.683	1.310	1.697	2.042	2.457	2.750
∞	0.253	0.674	1.282	1.645	1.960	2.326	2.576

This table is taken from Table III of Fisher and Yates: *Statistical Tables for Biological, Agricultural, and Medical Research*, published by Longman Group Ltd., London (previously published by Oliver and Boyd, Edinburgh).

見背面

題號： 327

科目：統計學(E)

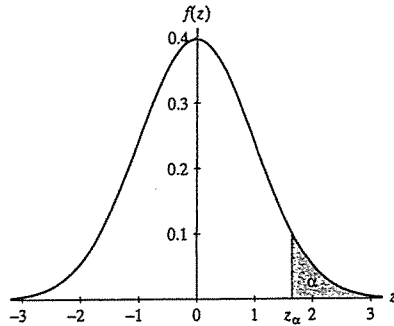
節次： 2

國立臺灣大學105學年度碩士班招生考試試題

題號：327

共 4 頁之第 4 頁

TABLE Vb: The Normal Distribution



$$P(Z > z_\alpha) = \alpha$$

$$P(Z > z) = 1 - \Phi(z) = \Phi(-z)$$

$z_\alpha$	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641
0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002

試題隨卷繳回