國立臺灣大學 105 學年度碩士班招生考試試題

1. Let X be a normal random variable with a population mean μ and variance σ^2. Let X_1, X_2, …, X_i be a random sample of size n from X. Please prove that the sample mean $\overline{X} = \frac{\sum_{i=1}^{n} X_i}{n}$ is an unbiased estimator of μ (**10 分**).

2. Let X be a normal random variable with a population mean μ and variance σ^2. Let X_1, X_2, …, X_i be a random sample of size n from X. Please prove that the sample variance $S^2 = \frac{\sum_{i=1}^{n} (X_i - \overline{X})^2}{n-1}$ is an unbiased estimator of σ^2 (**20 分**).

3. Let X be a normal random variable with a population mean μ and variance σ^2. Let X_1, X_2, …, X_i be a random sample of size n from X. Let $\overline{X}$ be the sample mean and $S_{\overline{X}}^2$ be the sample variance of $\overline{X}$. Please prove that the statistic $\frac{(\overline{X} - \mu)}{S_{\overline{X}}}$ is distributed as a *t*-distribution with n − 1 degrees of freedom (**20 分**).

4. Let $Y_i = \beta_0 + \beta_1 X_i + \varepsilon_i$, i = 1, 2, …, n. Assume ε_i are independently and identically distributed as normal distributions with population mean 0 and variance σ^2, and the independent variable X is measured without error. Please

   (1) derive the ordinary least squares estimators of β_0 and β_1 (**20 分**), and

   (2) show that both ordinary least squares estimators are linear unbiased estimators (**30 分**)

試題隨卷繳回