國立臺灣大學 105 學年度碩士班招生考試試題

1. (10 points) Let $X_1, X_2, \ldots$ be i.i.d. random variables and Y be a discrete random variable taking positive integer values. Assume that Y and X_i's are independent. Let $Z = \sum_{i=1}^{Y} X_i$.

   (a) Show that $E(Z) = E(Y) \cdot E(X_1)$. (5 points)

   (b) Show that $Var(Z) = E(Y) \cdot Var(X_1) + Var(Y) \cdot [E(X_1)]^2$. (5 points)

2. (15 points) The concentration (X) and the viscosity (Y) of a chemical product have a bivariate normal distribution with means (μ_X, μ_Y), standard deviations (σ_X, σ_Y), and a correlation coefficient ρ.

   (a) Define the moment-generating function. (5 points)

   (b) Please find the moment-generating function of $Z = X + Y$ and show that it has a normal distribution. (10 points)

3. (15 points) Let Z_1, Z_2, and Z_3 be independent standard normal distribution $N(0,1)$.

   (a) Please find the distribution of $W = \frac{Z_1}{\sqrt{(Z_2^2+Z_3^2)/2}}$. (5 points)

   (b) Please find the distribution of $W = \frac{Z_1}{\sqrt{(Z_1^2+Z_2^2)/2}}$. (10 points)

4. (10 points) Suppose that $X_1, \ldots, X_n$ are i.i.d. from the Pareto distribution $Pa(\alpha, \theta)$ with probability density function $f(x) = \theta\alpha^\theta x^{-(\theta+1)}$, $x > \alpha$, where $\alpha > 0$ and $\theta > 2$ are unknown parameters. Please use the method of moments to find the moment estimators of α and θ.

5. (20 points) Let $X_{(1)} < X_{(2)} < \cdots < X_{(n)}$ be the order statistics of a random sample of size n from a distribution with p.d.f.

$$f(x;\theta) = \frac{1}{\theta} e^{-\frac{x}{\theta}},\ 0 < x < \infty,$$

   where $0 < \theta < \infty$. Consider two functions of these order statistics S and R which are separately defined as follows:

$$S = \sum_{i=1}^{n} X_{(i)} \text{ and } R = \frac{nX_{(1)}}{\sum_{i=1}^{n} X_{(i)}}.$$

   (a) Show that S is a complete sufficient statistic for θ, and S and R are independent. (10 points)

   (b) Compute the expected value of R^{100}. (10 points)

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6. (20 points) Let $X_{(1)} < X_{(2)} < \cdots < X_{(n)}$ be the order statistics of a random sample of size n from a distribution with p.d.f.

$$f(x;\theta_1,\theta_2) = \frac{1}{2\theta_2},\ \theta_1 - \theta_2 < x < \theta_1 + \theta_2,$$

where $-\infty < \theta_1 < \infty$ and $\theta_2 > 0$.

(a) Show that $X_{(1)}$ and $X_{(n)}$ are joint complete sufficient statistics for θ_1 and θ_2. (10 points)

(b) Find the uniformly minimum variance unbiased estimators (UMVUEs) for θ_1 and θ_2. (10 points)

7. (10 points) Let $X_1, X_2, \ldots, X_n$ be a random sample of size n from a distribution with p.d.f.

$$f(x;\theta) = \theta^x(1-\theta)^{1-x},\ x = 0 \text{ or } 1,$$

where $0 < \theta < 1$. Consider the null hypothesis $H_0 : \theta = 0.5$ and the alternative hypothesis $H_1 : \theta < 0.5$, an investigator decides to reject H_0, if the observed sample sum $\sum_{i=1}^{n} x_i$ is less than or equal to a fixed constant c. Show that his/her decision rule is actually a uniformly most powerful test, and find the level of significance when $c = 1$.

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