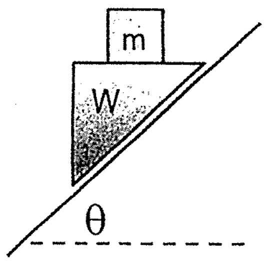
國立臺灣大學105學年度碩士班招生考試試題

1. (25%) A particle of mass m at rest starts to move from the origin in a straight line under the action of a variable force $F(t)$ as a function of time t in the following manner:

$$F(t)=\begin{cases} K/2, & 0<t<\lambda \\ K/4, & \lambda<t<2\lambda \\ K/8, & 2\lambda<t<3\lambda \\ \cdots & \\ K/2^n, & (n-1)\lambda<t<n\lambda \\ \cdots & \\ K/2^N, & (N-1)\lambda<t<N\lambda=T \end{cases}$$

where K, λ, and N are given constants and T is the total traveling time.
(a) Draw the plots for the particle's velocity and traveling distance as functions of time. (10%)
(b) Find the particle's final velocity at time T. (10%)
(c) Find the particle's overall distance from the origin at time T. (5%)

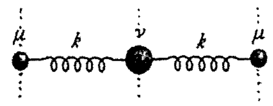
2. (25%) In a constant gravitational field g, consider a system of two wedges of masses m and W (shown in the plot) sliding down an inclined plane which is at an angle θ with respect to the horizontal ground. The wedge m is constrained to be always in contact with the wedge W and the contact surface is parallel to the ground. Assume that there is no dynamical friction force between the wedge W and the inclined plane, but there is a static friction force between the two wedges.



(a) Determine the accelerations of the two wedges. (10%)
(b) Find the static friction force between the two wedges. (10%)
(c) Find the apparent weight of the wedge m. (5%)

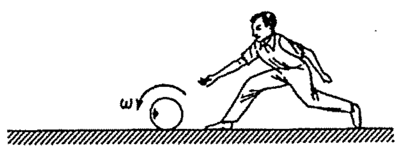
3. (25%) An idealized linear classical water molecule consists of three particles in a line connected by equal springs and constrained to move along the line joining them (shown in the plot). The outer two particles 1 and 3 have equal masses μ and the central one has mass ν, and the spring constant is k.

Equilibrium



(a) Find the normal modes (describe them) and normal frequencies. (10%)
(b) Write down the general solution. (10%)
(c) Write down the solution with initial conditions $x_1(0)=-A$, $x_2(0)=A\mu/\nu$, $x_3(0)=0$, and $\dot{x}_1(0)=\dot{x}_2(0)=\dot{x}_3(0)=0$. (5%)

4. (25%) A bowler releases his ball at a speed of 3 m/s. The ball has a diameter of 250 mm and its mass is 1.8 kg. The coefficient of friction between the ball and the floor is 0.1. Neglect wind resistance and rolling resistance.



(a) What spin ω should be put on the ball so there is no slipping? (10%)
(b) If the bowler puts on only half of this spin ω, keeping the same speed of 3 m/s, what is the final speed of the ball? (10%)
(c) In the case (b), what is the speed after 0.3 sec ? (5%)

試題隨卷繳回