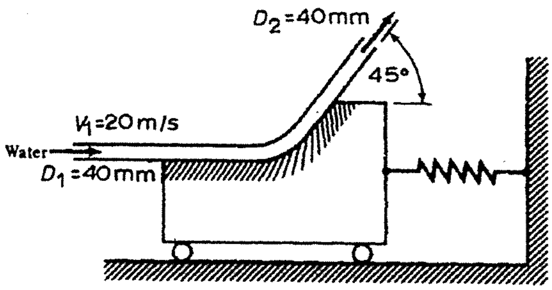
1. (25 %) A water jet of circular cross section with diameter 40 mm and velocity 20 m/s is deflected from horizontal to 45° upward by a cart on the ground and connected to a spring of stiffness $k = 1.6$ kN/m, as shown in the figure. Assume that the water jet cross section remains unchanged. Neglect the friction force between the water jet and the cart and between the cart and the ground. The mass density of water $\rho = 1000\,\text{kg/m}^3$.

(a) Calculate the volume flow rate Q (in unit of m^3/s) of the water jet. (7%)

(b) Based on (a), calculate the vertical component of the force F_y (in unit of N) on the jet. (8%)

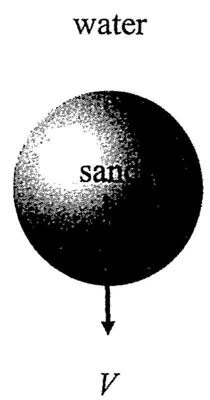
(c) Based on (b), calculate the compression length Δx (in unit of mm) of the spring. (10%)



2. (25%) The drag force on a sphere of diameter D moving at velocity V in a still fluid of dynamic viscosity μ is given by $F = 3\pi\mu VD$ provided that the Reynolds number is very small, that is, $\text{Re} \ll 1$. Consider a spherical sand grain of radius $20\mu m$ falling in water due to gravity. The mass density ratio of sand grain to water is 2.5. The kinematic viscosity ν of water is 10^{-2} cm^2/s. The gravitational acceleration is 9.8 m/s^2.

(a) Calculate the terminal velocity (in unit of m/hr) of the sand grain. (18%)

(b) Based on (a), calculate the Reynolds number Re. (7%)



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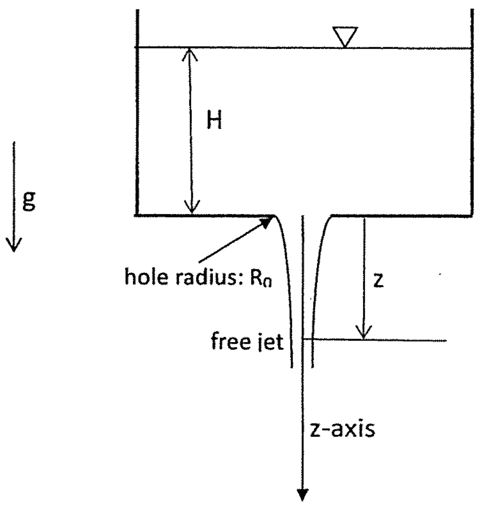
3. Liquid drains out of a large diameter tank through a circular opening of radius R_0 at the bottom of the tank in the form of a free-jet into the surrounding atmosphere. The tank diameter is much larger than $2R_0$. The height of the liquid free-surface is H. Assuming that the flow is inviscid. (Hint might be useful: $\nabla \cdot \mathbf{u} = \frac{1}{r}\frac{\partial(ru_r)}{r} + \frac{1}{r}\frac{\partial u_\theta}{\partial \theta} + \frac{\partial u_z}{\partial z}$)

(a) (9 pts) What is the speed of the jet $V(z)$ (as a function of z) at a distance z below the opening? State your assumptions.

(b) (8 pts) What is the radius of the jet $R(z)$ (as a function of z) at a distance z below the opening?

(c) (8 pts) Estimate the radial component of the fluid velocity within the jet.



4. Consider an ideal frictionless propeller windmill. The propeller is represented by an actuator disk which creates across the propeller plane a pressure discontinuity (see the pressure distribution below) of area A and local velocity. The upstream and downstream velocity of air are V_1 and V_2, respectively. Hint: You may need to draw the control volumes (CV1 and CV2) as in the figure below.

(a) (9 pts) Calculate the local wind velocity through propeller.

(b) (8 pts) Calculate the force exerted on the windmill support, given that the air density is ρ.

(c) (8 pts) What is the output power of the windmill?