國立臺灣大學 105 學年度碩士班招生考試試題

1. (10%) Find a family of curves which is the orthogonal trajectories of the curves: $x^2+(y-c)^2=c^2$, where c is an arbitrary constant.

2. (10%) The 2-D Laplacian operator in Cartesian coordinate is :

$$\nabla^2 \equiv \left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right).$$

Transform this operator in Polar coordinate (r,θ) with $x=r\cos\theta\,;\,y=r\sin\theta$

3. (20%) Find the complete solution of the following nonhomogeneous matrix equation:

$$x_1'=x_1-x_2+\frac{e^{-t}}{1+t^2}$$

$$x_2'=2x_1-2x_2+\frac{2e^{-t}}{1+t^2}$$

4. (20%) Find the inverse Laplace transform of the function $Y(s)=\dfrac{2s^2-s}{(s^2+4)^2}$

5. (20%) Consider the following eigenvalues problem of $y(x)$:

$$y''+\lambda^2 y=0 \quad -p\geq x\leq p$$

$$B.C.\ \begin{cases} y(-p)=y(p) \\ y'(-p)=y'(p) \end{cases}$$

if λ_m and λ_n are two distinct eigenvalues of the problem, show that the corresponding eigenfunctions $y_m(x)$ and $y_n(x)$ are orthogonal in $(-p,p)$.

6. (20%)What is the Fourier expansion of the periodic function

$$f(t)=4-t^2 \qquad -2\leq t\leq 2$$

And also prove that $1-\dfrac{1}{2^2}+\dfrac{1}{3^2}-\dfrac{1}{4^2}+\dfrac{1}{5^2}-\ldots=\dfrac{\pi^2}{12}$

試題隨卷繳回