1. *(20%) Find the characteristic polynomial, the eigenvalues and the eigenvectors of the following matrix:*

$$\mathbf{A} = \begin{bmatrix} 2 & 1 & -1 \\ 3 & 2 & -3 \\ 3 & 1 & -2 \end{bmatrix}$$

2. *(20%) Prove the following vector theorem:*

if $\bar{\nabla} \cdot \bar{u} = 0$

and $\bar{\nabla} \times \bar{u} = \bar{\omega}$

then $\nabla^2 \bar{u} = \bar{\nabla} \cdot \bar{\nabla} \bar{u} = -\bar{\omega}$ *and write out in Cartesian components if*

$\bar{u} = u_1 \bar{i} + u_2 \bar{j} + u_3 \bar{k}$ *and* $\bar{\omega} = \omega_1 \bar{i} + \omega_2 \bar{j} + \omega_3 \bar{k}$

Explain the physical meanings in fluid mechanics.

(Hint: use the vector identity of vector triple products

$\bar{A} \times (\bar{B} \times \bar{C}) = (\bar{A} \bullet \bar{C})\bar{B} - (\bar{A} \bullet \bar{B})\bar{C}$ *)*

3. *(20%) Solve the following initial value problem by the Laplace transform method; plot the input, output graphs and also interpret the physical meanings in hydrology :*

$y'' + 3y' + 2y = \delta(t-1) - \delta(t-2)$

$y(0) = y'(0) = 0$

where δ *is the Dirac delta function.*

4. *(20%) Find the Fourier series expansion of the following period function* $f(x)$*:*

$$f(x) = \begin{cases} x - 2L & 2L \le x \le 3L \\ x - 4L & 3L \le x \le 4L \end{cases}$$

and $f(x + 2L) = f(x)$.

5. *Solve the following partial differential equation in a square cavity by using separation of variables and Fourier series expansion method*

$$\nabla^2 u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \qquad 0 \le x \le 1 \quad \text{and} \quad 0 \le y \le 1.$$

(a) (10%) Subject to the boundary conditions (BCs) :

u(x,0) = u(0,y) = u(1,y) = 0 and u(x,1) = 1

what is the name of this equation, and where it appears in engineering?

(b) (10%) Subject to the BCs: u(0,y) = u(x,0)= 0 and u(x,1) = u(1,y) = 1.

(Hint: use the superposition principle)

試題隨卷繳回