國立臺灣大學 105 學年度碩士班招生考試試題

※ 注意：請於試卷內之「非選擇題作答區」標明題號依序作答。

1. (10%) Let $X_1, \ldots, X_n$ be a random sample from a continuous distribution $F(x)$. Derive the distribution of the ith order statistic of $X_{(1)}, \ldots, X_{(n)}$.

2. (7%) (8%) Let $T \sim t_\nu$, $Z \sim N(0,1)$, and $X_i \sim \chi^2_{\nu_i}$, $i = 1, 2$, with X_1 and X_2 being mutually independent. Derive the limiting distribution of T as $\nu \to \infty$ and the distribution of $X_1/(X_1 + X_2)$.

3. (5%) (5%) Let the distribution of U conditioning on $T = t$ be $Uniform(0, t)$ and T follow an exponential distribution with rate $\lambda > 0$. Compute the expectation and variance of U.

4. (5%) (10%) Let $X_1, \ldots, X_n$ be a random sample from $N(\mu, \sigma^2)$ and $\Phi(\cdot)$ stand for the standard normal distribution. Find the maximum likelihood estimator of $\Phi((x - \mu)/\sigma)$ for a given value x and derive its asymptotic distribution.

5. (10%) (10%) Let $X_1, \ldots, X_n$ be a random sample from a density function $f_X(x|\theta) = \theta x^{\theta-1} I(0,1)(x)$, $0 < \theta < \infty$. Find the uniformly minimum variance unbiased estimator (UMVUE) of θ and derive the asymptotic distribution of this UMVUE.

6. (15%) Let $X_1, \ldots X_n$ be a random sample from $N(\theta, \sigma^2)$. Find an unbiased size α test for the hypotheses $H_0 : \theta_1 \le \theta \le \theta_2$ versus $H_A : \theta < \theta_1$ or $\theta > \theta_2$.

7. (15%) Let $X_1, \ldots, X_n$ be a random sample from $N(\theta, \sigma^2)$ with σ^2 being unknown. Derive the power function of the size α likelihood ratio test for the null hypotheses $H_0 : \theta \le \theta_0$ versus $H_A : \theta > \theta_0$.

試題隨卷繳回