國立臺灣大學 104 學年度碩士班招生考試試題

1. (30%) A company can produce three types of chocolate bars. Each chocolate bar contains sugar and chocolate. The usage of these two ingredients for each type of chocolate bar and the profit earned from each chocolate bar are summarized in Table 1. The company has 50 oz of sugar and 100 oz of chocolate every week. Define xi to be the number of type i chocolate bars manufactured. The weekly production plan can be formulated as the following linear programming.

Table 1

Chocolate Bar	Sugar (oz)	Chocolate (oz)	Profits (cents)
1	1	2	3
2	1	3	7
3	1	1	5

$$\begin{aligned} \max\ & z = 3x1 + 7x2 + 5x3 \\ s.t.\ & x1 + x2 + x3 \le 50 \quad \text{(sugar)} \\ & 2x1 + 3x2 + x3 \le 100 \quad \text{(chocolate)} \\ & x1, x2, x3 \ge 0 \end{aligned}$$

By adding two slack variables $s1$ and $s2$ accordingly, the optimal tableau is obtained as follows. Answer the following questions based on the given optimal tableau.

	z	$x1$	$x2$	$x3$	$s1$	$s2$	rhs
z	1	3	0	0	4	1	300
$x3$	0	1/2	0	1	3/2	-1/2	25
$x2$	0	1/2	1	0	-1/2	1/2	25

(a) (5%) For what values of type 1 chocolate bar profit does the current basis remain optimal?

(b) (5%) If the profit for a type 2 chocolate bar were 13 (cents), what would be the new optimal solution to the company?

(c) (5%) For what amount of available sugar would the current basis remain optimal?

(d) (5%) If 60 oz of sugar will be available, what would be the new profit? How many of each chocolate bar should the company make?

(e) (5%) The company is considering to produce a new chocolate bar which requires 3 oz of sugar and 4 oz of chocolate and earns 17 cents. Should this new product be manufactured? Justify your answer.

(f) (5%) If the company wants to add butter to chocolate bars, types 1, 2, and 3 require 1, 2, and 1 oz, respectively. The company can acquire 70 oz of butter and assume that the profit earned from each type of chocolate bar remains the same. How does the profit of the company change (increase, decrease or remain the same)? Why?

2. (10%) For a problem formulated in linear programming, its optimal solution will satisfy three properties: primal feasibility, dual feasibility, and complementary slackness. Before reaching an optimal solution,

(a) (5%) how are these three properties maintained in the process of applying Simplex method?

(b) (5%) how are these three properties maintained in the process of using the Hungarian algorithm to solve an assignment problem?

3. (10%) Suppose as many as 200 cars per hour can travel between any two of the cities A, B, and C. The maximum number of cars travels from city A to city C in the next two hours can be determined by using linear programming (LP). Set up an LP model for the problem.

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