國立臺灣大學 104 學年度碩士班招生考試試題

1. (25 %) A mass M slides without friction on the roller coaster track shown in Fig. 1. The curved sections of the track have radius of curvature R. The mass begins its descent from the height h. At some value of h, the mass will begin to lose contact with the track. (Note: In Fig. 1, A is the inflection point of the track)
   (a) Before the inflection point A, find the normal reaction of the track on the mass. (7 %)
   (b) After the inflection point A, find the normal reaction of the track on the mass. (7 %)
   (c) Calculate the minimum value of h for which the mass will begin to lose contact with the track. (11 %)

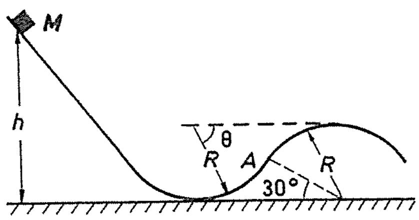

Figure 1

2. (25%) The end of a chain of length L and mass ρ per unit length which is piled on a platform is lifted vertically with a constant velocity v by a variable force P (see Fig. 2).
   (a) Find P as a function of the height x of the end above the platform. (15%)
   (b) Find the energy lost during the lifting of the chain. (10%)

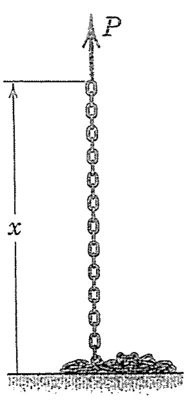

Figure 2

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3. (25%)

(1) State the defining property for a body to be rigid. (5%)

(2) Let A, B be two points in a rigid body, cf. Fig. 3, and $\mathbf{r}^A$, $\mathbf{r}^B$ denote the corresponding position vectors. Show that the velocity of A can be expressed as

$$\dot{\mathbf{r}}^A = \dot{\mathbf{r}}^B + \boldsymbol{\omega} \times \mathbf{r}^{BA},$$

where $\boldsymbol{\omega}$ is the angular velocity of the rigid body, and $\mathbf{r}^{BA} = \mathbf{r}^A - \mathbf{r}^B$. (10%)

(3) Consider the general motion of a rigid body for which the angular velocity $\boldsymbol{\omega}$ and angular acceleration $\dot{\boldsymbol{\omega}}$ are both nonzero and are not parallel.

(i) If the point A in the body has zero velocity, find the other points in the rigid body which also has zero velocity; (5%)

(ii) If the point A in the body has zero acceleration, show that there is no other point which also has zero acceleration. (5%)

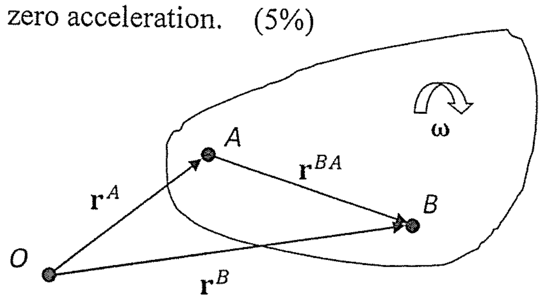

Figure 3

4. (25%) Consider a circular rigid cylinder of radius a and mass m moving on a rough horizontal plane, as shown in Fig. 4. The position of the center of the cylinder is denoted by (x, y), with x in the horizontal direction and y in the vertical direction. Let the angle of rotation of the cylinder be denoted by φ. The cylinder is pushed by a horizontal force with the magnitude P at the height h. Denote the coefficient of static friction between the cylinder and the surface by μ. Let the tangential resistive force acting on the cylinder be denoted by T. The moment of inertia of the cylinder is assumed to be $(1/2)ma^2$.

(1) Write down the general equations of motion for x and φ, in terms of P and T. (9%)

(2) Determine the condition that the cylinder rolls without sliding on the surface. Under that condition, find the magnitude of the resistive force T, and determine the height h such that $T = 0$. (8%)

(3) Determine the condition on P and h such that the cylinder rolls as well as slides on the horizontal plane. In that case, find the acceleration of the center of the cylinder, i.e. $\ddot{x}$. (The coefficient of kinetic friction is denoted by μ_k.) (8%)

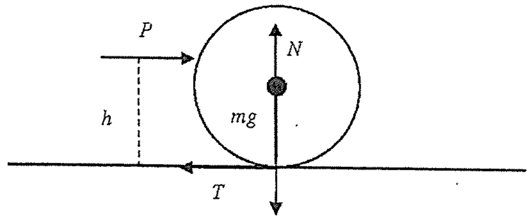

Figure 4