

※ 注意：請用 2B 鉛筆作答於答案卡，並先詳閱答案卡上之「畫記說明」。

1. (5%) A function $y(x)$ satisfies the following

$$(y')^2 = 9x^4y, \quad y(0) = 0.$$

Find the possible values of $y(2)$ from below

- (A) 0
 - (B) -4
 - (C) 16
 - (D) -64
 - (E) None of the above
2. (5%) For the following linear differential equation

$$x^3y''' + 2x^2y'' + xy' = y, \quad x > 0.$$

Choose from the below set of independent solutions of the above equation.

- (A) $\{x, x \sin x, x \cos x\}$
 - (B) $\{x, x \sin 3x\}$
 - (C) $\{x, \sin \ln x\}$
 - (D) $\{x, \sin \ln 3x\}$
 - (E) None of the above
3. (5%) Consider the following differential equation of a function $y(x)$

$$y^{(4)} + 4y = 0.$$

Choose from the below possible solutions of the above equation.

- (A) $y(x) = e^{2x}$
 - (B) $y(x) = e^{-x} \cos x$
 - (C) $y(x) = e^{-\sqrt{2}x} \sin x$
 - (D) $y(x) = e^x \cos(x + \pi/4)$
 - (E) None of the above
4. (5%) A function $y(x)$ satisfies the following:

$$y'' + 4y' + 4y = 4e^{-2x}$$

Choose from the below possible solutions of the above equation.

- (A) $y(x) = (x + 2x^2)e^{-2x}$
 - (B) $y(x) = e^{-2x}$
 - (C) $y(x) = (1 + 2x + x^2 + x^3)e^{-2x}$
 - (D) $y(x) = (1 + x)(1 + 2x)e^{-2x}$
 - (E) None of the above
5. (5%) A dynamical system with a time-varying state variable $x(t)$ is described by the following differential equation

$$(t^2 + t - 2)x'' - 2(2t + 1)x' + 6x = 0, \quad t \geq 0$$

Under the conditions

$$0 < x(1) < 1, x'''(1) = 0$$

Find the possible values of $x(2)$ from below,

- (A) 1
- (B) 2
- (C) 4
- (D) 8
- (E) None of the above

6. (5%) Which of the following function is of exponential order?
 (A) $5e^{4t}$
 (B) $\cosh 3t$
 (C) $\cosh t^4$
 (D) $\sin e^{t^2}$
 (E) None of the above
7. (5%) Evaluate the Laplace or inverse Laplace transform of the following functions. Which are correct?
 (A) $f(t) = e^{-t} \times H(t - 5)$, $L\{f(t)\} = (1 - 5e^{-t})H(t - 5)$
 (B) $f(t) = \int_0^5 H(\tau - t)d\tau$, $L\{f(t)\} = (5 - t)H(5 - t)$
 (C) $f(s) = \frac{1}{s} \tanh s$, $L^{-1}\{f(s)\} = H(t) - 2H(t - 1) + 2H(t - 3) - \dots$
 (D) $f(s) = \frac{e^{-s}}{s^2 + s + 1}$, $L^{-1}\{f(s)\} = \frac{2}{\sqrt{3}}H(t - 1)e^{1-t/2} \sin 2\sqrt{3}(t - 1)$
 (E) None of the above
8. (5%) The inverse Laplace transform of function $f(s) = \ln \left(\frac{s^2 + 1}{s^2 + s} \right)$ is expressed as

$$f(t) = \frac{A(t) \cos t + B(t) + e^{C(t)}}{D(t)}$$

Which of the followings are correct?

- (A) $A(t) = -2t$
 (B) $B(t) = 1$
 (C) $C(t) = -t$
 (D) $D(t) = \sin t$
 (E) None of the above
9. (5%) Given a forcing function

$$f(t) = \begin{cases} 1, & 0 \leq t < 1, 2 \leq t < 3, 4 \leq t < 5, \dots \\ 0, & 1 \leq t < 2, 3 \leq t < 4, 5 \leq t < 6, \dots \end{cases}$$

Solve the differential equation $x'' + x = f(t)$, with that $x(0) = x'(0) = 1$, and we have

$$x(t) = \sin A(t) + B(t) \cos C(t) + 2t + D(t)H(t - 2) + E(t)H(t - 4) + \dots$$

Which of the followings are correct?

- (A) $A(t) = t$
 (B) $B(t) = -1$
 (C) $D(t) = 4(\cos(2 - t) - 1)$
 (D) $x(5) > 4$
 (E) None of the above

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10. (5%) Evaluate the Fourier or inverse Fourier transform of the following functions. Which are correct?

(A) $f(\omega) = H(\omega + a) - H(\omega - a), a > 0, F^{-1}\{f(\omega)\} = \sin x/ax$

(B) $f(x) = 4x^2e^{-3|x|}, F\{f(x)\} = 48 \left[\frac{1}{(\omega^2+9)^2} - \frac{4\omega^2}{(\omega^2+9)^3} \right]$

(C) $f(\omega) = \frac{4 \sin \omega}{\omega} - \frac{1}{\sqrt{|\omega|}}, F^{-1}\{f(\omega)\} = 2[H(x+1) - H(x-1)] - \frac{1}{\sqrt{2\pi x}}$

(D) $f(x) = \frac{\cos 3x}{x^2+2}, F\{f(x)\} = \frac{\sqrt{2\pi}}{4} (e^{-\sqrt{2}|\omega-3|} + e^{-\sqrt{2}|\omega+3|})$

(E) None of the above

11. (10%) Which of the following statements is NOT the probability axiom? (A) For any event A, $P[A] \geq 0$. (B) For any events A and B, $P[A \cup B] = P[A] + P[B] - P[A \cap B]$. (C) Let S be the sample space. $P[S] = 1$. (D) If A and B are mutually exclusive, $P[A \cup B] = P[A] + P[B]$, (E) None of the above.

12. (10%) Let X be a continuous random variable and its pdf is given as $a * e^{-2x}$ for $x=0$ to 10. The value of a should be (A) 1/2, (B) $1/(1-e^{-20})$, (C) $(1-e^{-20})/2$, (D) 2, (E) None of the above.

13. (10%) Let X be the number of Route 1 buses arriving at NTU station within t hours, and the PMF of X be $(5t)^x e^{-5t}/x!$ for $x=0, 1, 2, \dots$. Route 1 buses leaving from NTU may take an express or local route, depending on the bus driver. Assume that each driver takes the express route with a probability of 1/3 (independently). What would be the average waiting time for Peter who needs to take an express Route 1 bus at NTU? (A) 36 minutes, (B) 30 minutes, (C) 15 minutes, (D) 12 minutes, (E) None of the above.

14. (10%) Let Y be the number of Route 2 buses arriving at NTU station within t hours, and the PMF of Y be $(t)^y e^{-t}/y!$ for $y=0, 1, 2, \dots$. The arrival processes of Route 1 and 2 buses are independent. Assume that Peter can take Route 2 or any Route 1 bus. What is the average waiting time for Peter at NTU? (A) 6 minutes, (B) 8 minutes, (C) 10 minutes, and (D) 15 minutes, (E) None of the above.

15. (10%) X and Y are two random variables with the probability density functions (pdf) shown in the figures below. The probability of $X+Y=0$ is (A) 0, (B) 0.2, (C) 0.5, (D) The information is not sufficient to calculate the answer, (E) None of the above.

