

1. Please solve the following equation (15%)

$$x^4 \frac{d^2 y}{dx^2} + y = 0 \quad (\text{Hint: } x = \frac{1}{t})$$

2. Please find the inverse Laplace transform of  $L^{-1} \left\{ \frac{s^2 + 6s - 8}{[(s+2)^2 + 16]^2} \right\}$  (20%)

3. Please (a) identify the rank (5%) and (b) determine the inverse of the following matrix (10%)

$$A = \begin{bmatrix} 0 & -0.3 & 0.75 \\ 0.4 & 2 & 3 \\ 0 & 0 & 8 \end{bmatrix}$$

4. Please solve the following differential equations by using eigenvalues and eigenvectors to obtain the general solution. (15%)

$$y_1'' = -11y_1 + 3y_2$$

$$y_2'' = 12y_1 - 6y_2$$

5.  $f(x) = x$ ,  $0 < x < 2$  (a) please expand in a half-range Fourier sine series (15%);  
(b) find a Fourier cosine series for  $f(x) = x^2$  using your solution from (a) (10%);  
(c) using your solution from (b) to evaluate the series  $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^2}$  (10%).

試題隨卷繳回