國立臺灣大學 103 學年度碩士班招生考試試題

請詳細列出計算及推導過程，否則不予計分。題目前中括號 [ ] 內之數字為該題配分。

1. [25 points] Define:

   (a) Random experiment, probability set function and random variable.

   (b) Method of least squares and method of maximum likelihood.

   (c) Sufficiency, consistency and efficiency.

   (d) Type I error, type II error and power function.

   (e) Uniformly most powerful test and likelihood ratio test.

2. [5 points] State the Central Limit Theorem.

3. [10 points] State the Rao-Blackwell Theorem.

4. [10 points] State the Neyman-Pearson Theorem.

5. [25 points] Let X and Y be independent exponential random variables, with respective pdf's

$$f(x|\lambda) = \frac{1}{\lambda}e^{-x/\lambda},\ x > 0, \quad f(y|\mu) = \frac{1}{\mu}e^{-y/\mu},\ y > 0$$

   Instead of X and Y, we observe the random variables Z and W, where

$$Z = \min(X, Y) \quad \text{and} \quad W = \begin{cases} 1 & \text{if } Z = X, \\ 0 & \text{if } Z = Y. \end{cases}$$

   (a) Find the joint distribution of Z and W.

   (b) Prove that Z and W are independent.

   (c) Assume that (Z_i, W_i), $i = 1, \ldots, n$ are n i.i.d. observations. Please find the Maximum MLEs of λ and μ.

6. [15 points] Let $X_1, \ldots, X_n$ be i.i.d. from Normal distribution $N(\mu, \sigma^2)$ with unknown mean μ and variance σ^2.

見背面

(a) We are interested in the estimation of the percentile η such that $P(X_1 \leq \eta) = p$ with a fixed $p \in (0,1)$. Please find the UMVUE (uniformly minimum variance unbiased estimator) for η.

(b) If σ^2 is known, please find the UMVUE of μ^2 and the Cramér-Rao Lower Bound for the variance of unbiased estimator of μ^2.

7. [10 points] Let $X_1, \ldots, X_n$ be i.i.d. from the Uniform distribution $U(\theta, \theta+1)$, $\theta \in R$. Suppose that $n \geq 2$. Show that a UMP (uniformly most powerful) test of size α for testing $H_0 : \theta \leq 0$ versus $H_1 : \theta > 0$ is of the form

$$T(X_{(1)}, X_{(n)}) = \begin{cases} 0 & \text{if } X_{(1)} < 1 - \alpha^{1/n} \text{ and } X_{(n)} < 1, \\ 1 & \text{otherwise.} \end{cases}$$

試題隨卷繳回