(Problem 1, 20%) *NTUOR* is a book store that sells OR textbooks to NTU students. The list price of the OR textbook is $1,300 per copy, and the demand for the OR textbook is uncertain.

I. At the beginning of this semester, *NTUOR* can order 60, 80, or 100 copies of the book from the publisher. The publisher offers quantity discount. For *NTUOR*, the ordering costs under different ordering quantities are listed in the following table.

# of books ordered	60	80	100
Ordering costs	$61,000	$77,000	$91,000

II. *NTUOR* can either sell the book at the list price ($1,300 per copy) or offers 10% discount ($1,170 per copy). The demand distributions under different prices are listed in the following tables. (For example, if *NTUOR* sells the books at $1,170, there is a 0.4 probability that the demand is 80.)

The demand distribution (Price = $1,300)

Demand	Probability
70	0.6
90	0.4

The demand distribution (Price = $1,170)

Demand	Probability
80	0.4
100	0.6

III. Any unmet demand for the textbook will be lost. (For example, if *NTUOR* orders only 60 books from the publisher and the demand is 80, *NTUOR* can sell only 60 books.)

There are two decision variables in this decision problem: ordering quantity and price.

(a) (5%) Create a decision tree model for this decision problem.
(b) (10%) Solve your decision tree model and find the optimal ordering quantity and price for *NTUOR*.
(c) (5%) If *NTUOR* can return unsold textbooks to the publisher for a refund of $800 per copy, find the optimal ordering quantity and price for *NTUOR*.

(Problem 2, 15%) NTU computer center uses two identical computers to server two job types, internal jobs and external jobs. Both types of jobs follow Poisson arrival processes.

I. The arrival rate of internal jobs is 18 per hour.
II. The arrival rate of external jobs is 15 per hour.
III. The service time for a job is an exponential random variable with mean 3 minutes. (Internal jobs and external jobs have the same service time distribution.)

(a) (7%) When one computer is used exclusively for internal jobs and the other for external jobs, find the average waiting time for both job types.
(b) (8%) When two computers handle both types of jobs, find the average waiting time. (In this case, there is only one queue and both computers can handle both types of jobs.)

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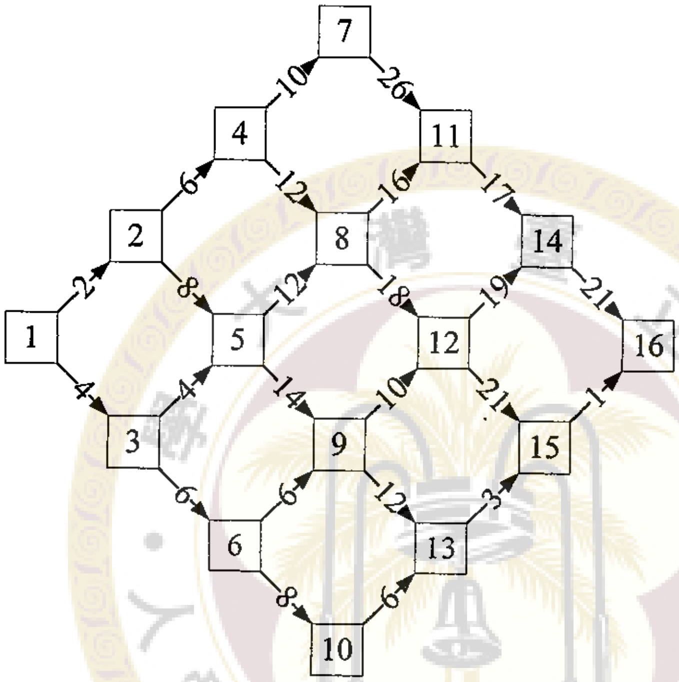
(Problem 3, 15%) In the following network,

I. The length of each arc is marked on the arc. (For example, the distance between node 2 and node 4 is 6.)

II. All arcs are one-way arcs. (For example, you are allowed to move from node 2 to node 4 but not allowed to move from node 4 to node 2.)



(a) (15%) Find the shortest distance path from node 1 to node 16.

(Problem 4, 15%) Consider the linear program (primal problem)

$$\begin{array}{lrcrcrcrcr} \text{maximize} & 2x_1 & + & x_2 & + & 5x_3 & - & 3x_4 & & \\ \text{subject to} & x_1 & + & 2x_2 & + & 4x_3 & - & x_4 & \le & 6 \\ & 2x_1 & + & 3x_2 & - & x_3 & + & x_4 & \le & 12 \\ & x_1 & & & + & x_3 & + & x_4 & \le & 4 \\ & x_1, & & x_2, & & x_3, & & x_4 & \ge & 0 \end{array}$$

Let x_5, x_6, x_7 be the slack variables for the first, second, and third constraints. The dual problem is given by

$$\begin{array}{lrcrcrcr} \text{minimize} & 6y_1 & + & 12y_2 & + & 4y_3 & & \\ \text{subject to} & y_1 & + & 2y_2 & + & y_3 & \ge & 2 \\ & 2y_1 & + & 3y_2 & & & \ge & 1 \\ & 4y_1 & - & y_2 & + & y_3 & \ge & 5 \\ & -y_1 & + & y_2 & + & y_3 & \ge & -3 \\ & y_1, & & y_2, & & y_3 & \ge & 0 \end{array}$$

Let y_4, y_5, y_6, y_7 be the surplus variables for the first, second, third, and fourth constraints, respectively, in the dual program.

(a) (5%) Write all of the complementary slackness conditions for the primal and dual problems.

(b) (10%) Given the basic feasible solution with basic variables, $x_1 = 4$, $x_2 = 1$, $x_6 = 1$, use complementary

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slackness to find the complementary dual basic solution ($y_1, y_2, y_3, y_4, y_5, y_6, y_7$).

(Problem 5, 16%) You are asked to plan producing at least 2000 units of components on three machines. The minimum lot size on any machine is 500 units of components. Let x_j be the number of components produced on machine *j*, *j* = 1, 2, 3. $y_j = 1$ if machine *j* is used and 0 otherwise. The fixed running costs, unit production costs, and capacities of each machine are listed as follows:

Table 1: The estimated costs and capacities of machines (Scenario 1)

Machine	Fixed running cost	Unit production cost	Capacity (units)
1	300	2	600
2	100	10	800
3	200	5	1200

(a) (4%) Formulate the constraints that the minimum lot size on any machine is 500 units of components.
(b) (4%) Formulate the constraint that the production quantity is at least 2000 units of components.
(c) (4%) Formulate the constraints of the capacity of each machine.
(d) (4%) Formulate the objective function to minimize the total production cost.

(Problem 6, 19%) Following **Problem 5**, suppose that the fixed running cost is difficult to estimate. Another possibility (Scenario 2) of the fixed running cost of machines 1, 2, and 3 is 200, 150, and 300, respectively and other parameters in Table 1 remain the same. The new demand of needed components is 2500 units. The number of components produced on machine *j* varies in various scenarios, but the binary variables, y_j's, are independent of different scenarios. Let $x_{j,\omega}$ be the number of components produced on machine $j = 1, 2, 3$ under Scenario $\omega, \omega = 1, 2$. Let us determine a fixed set of y-variables that we call a robust solution, to be evaluated in every scenario ω. Let L_ω represent the optimal objective function value under Scenario ω determined by fixing the y-variable values to the robust solution and solving for x-variables. Let O_ω^* be the minimum total cost under Scenario ω determined by solving for x- and y-variables.

(a) (4%) Formulate the constraints that the minimum lot size on any machine is 500 units of components under Scenario 1 and Scenario 2.
(b) (4%) Formulate the constraint that the production quantity is at least 2500 units of components under Scenario 2.
(c) (3%) (TRUE or FALSE) O_ω^*'s are always less than or equal to L_ω's under Scenario 1 and Scenario 2.
(d) (4%) Formulate the associated constraints to let δ be the maximum difference between L_ω and O_ω^* across Scenario 1 and Scenario 2. (Hint: δ needs to be greater than or equal to the difference between L_ω and O_ω^* across Scenario 1 and Scenario 2.)
(e) (4%) Formulate the objective function to minimize the maximum difference between L_ω and O_ω^* across Scenario 1 and Scenario 2.