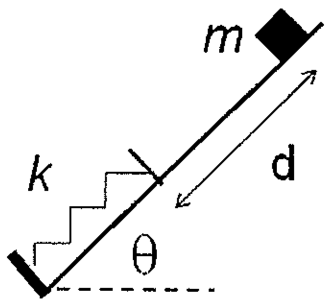
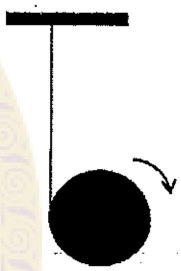
1. In a constant gravitational field $g$, consider a particle of mass $m$ sliding on a smooth inclined plane at an angle $\theta$ with respect to the horizontal ground. At the lower position there is a massless spring with the spring constant $k$. Initially the particle is released from rest and is at a distance $d$ from the spring along the inclined plane. Neglect any friction effect during the motion. Once the particle hits the spring, answer the following questions. (25%)




(a) Determine the equation(s) of motion for the particle.
(b) Find the velocity when the particle's acceleration is zero.
(c) Find the maximum compressed distance of the spring.

2. In a constant gravitational field $g$, consider a homogeneous rigid cylinder of mass $M$ and radius $R$ hanging from the ceiling with a massless inextensible string wrapped around the cylinder's periphery. Initially the cylinder is released from rest and rotates as the string unwinds. The moment of inertia of the cylinder about its center of mass is assumed to be $(1/2)MR^2$. (25%)

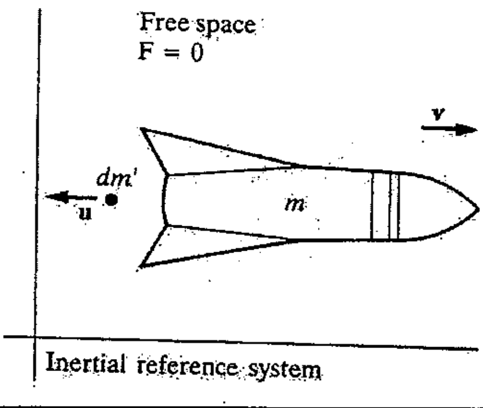


(a) Determine the equations of motion for the cylinder.
(b) Find the linear acceleration of the center of mass of the cylinder and the angular accelerations of the cylinder in terms of $g$, $M$, and $R$.
(c) Find the tension on the string in terms of $g$, $M$, and $R$..

3. Consider a rocket with mass $m$ moving in a one-dimensional free space with velocity $v$, cf. the attached figure. During an infinitesimal period $dt$, the mass (exhaust) $dm'$ is ejected from the rocket with the relative ejection velocity $u$. Let the infinitesimal velocity gained by the rocket be denoted by $dv$. (25%)
(a) By applying the law of conservation of linear momentum, find the relation between $dm', u, m, dv$, with higher order terms in $dm'$ and $dv$ being neglected.
(b) Note that $dm' = -dm$, where $dm$ is the infinitesimal change of the mass $m$. Show that the relation between $dm$ and $dv$ is

$$dv = -\frac{udm}{m}$$

(c) Integrate the above equation to find the relation between the mass and the velocity. In particular, let the masses of the rocket before and after ejection be represented by $m_1$ and $m_2$, respectively ($m_1 > m_2$). Find the velocity gained by the ejection process.

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4. In a constant gravitational field g, consider the motion of a rigid cylinder of radius $a$ and mass $m$ rolling down along an inclined plane with inclination angle $\theta$ (>0), as shown in the following figure. Let the angle of rotation of the cylinder be denoted by $\phi$. The interactive force between the cylinder and the surface may be decomposed into two parts: the force along the inclined plane, $F_p$, and the normal force, $F_N$. Let the coefficient of the static and the sliding friction between the cylinder and the inclined plane be denoted by $\mu_s$ and $\mu_k$, respectively. Assume that the contact persists during the motion. (25%)

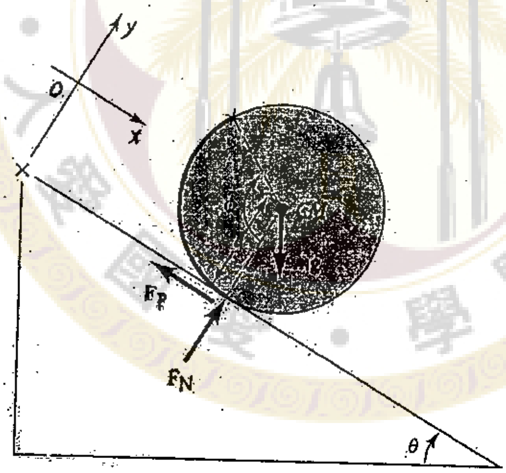
(a) Write down the equations of motion, including the translation of the center of mass and the rotational motion. (The moment of inertia of the cylinder about its center of mass is assumed to be $(1/2)ma^2$.)

(b) Determine $F_N$.

(c) Assume that the contact of the cylinder and the inclined plane is perfectly rough so that no slipping can occur. Find the acceleration of the center of mass along the inclined plane.

(d) Determine the condition on the angle $\theta$ such that the cylinder rolls without sliding along the inclined plane.

(e) If the cylinder rolls as well as slides along the inclined plane, find the acceleration of the center of mass along the inclined plane and the angular acceleration.

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