國立臺灣大學 103 學年度碩士班招生考試試題

※ 注意：請於試卷內之「非選擇題作答區」作答，並應註明作答之題號。

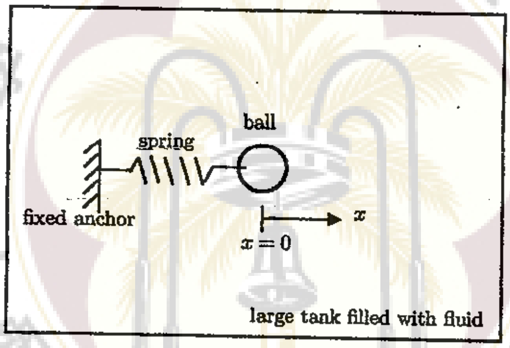
1. (25 %) A rigid spherical ball with a mass, m_b, and radius, R_b, is attached to a massless frictionless spring with spring constant, k, in a large tank filled with fluid with density, ρ. Neglect gravity. The spring and the ball are considered to move in the x-direction only (see figure below). Any flow disturbance due to the spring or its anchor is negligible. The ball is displaced from its equilibrium position $x = 0$ to $x = A$ and released, and subsequently oscillates.

(a.) (6%) List the forces acting on the ball.

(b.) (7%) Now assume that any fluid flow in the tank is irrotational. What is the frequency with which the ball oscillates? Your final answer should contain only the following parameter: m_b, R_b, ρ, k.

(c.) (6%) Let the Reynolds number be defined as: $Re=U_{max}R_b/\nu$, where U_{max} is the maximum velocity of the ball and ν is the kinematic viscosity of the fluid. When Re << 1, do you expect the analysis of (b) to be applicable? Why or why not?

(d.) (6%) When Re >> 10,000, do you expect the analysis of (b) to be applicable? Why or why not?



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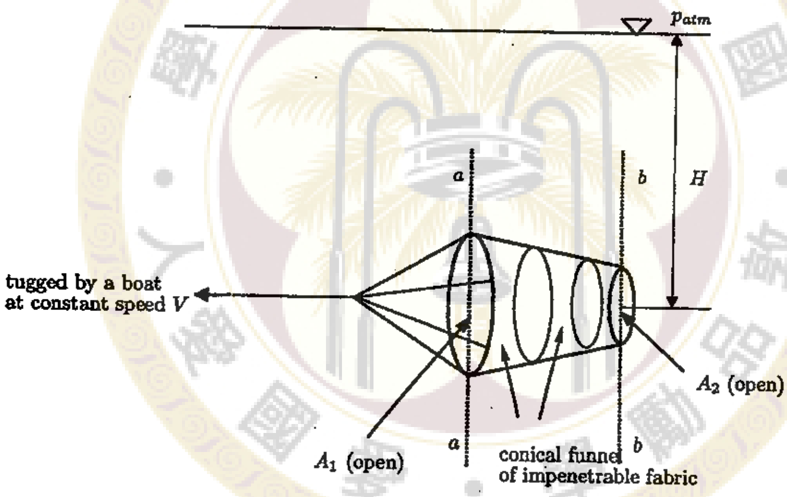
2. (25%) A boat is tugging the 'device' shown in the sketch at a constant speed V. The device consists of a conical funnel whose surface is made from an impenetrable fabric. The device is connected by a rope to the tugging boat. The mean water depth is H all along the device. The inlet area of the funnel is A_1, which narrows and becomes A_2 at the exit.

(a.) (8%) Choose a reference frame suitable for analyzing the water flow through the 'funnel'. In this reference frame carefully draw the streamlines of the flow. Be sure to include the region ahead of funnel inlet, and that just downstream from the funnel exit. Also show velocity profiles that you expect at the funnel inlet (along line a-a) and funnel exit (along the line b-b)

Take the flow to be incompressible and inviscid

(b.) (4%) What is the static pressure p_2 at the funnel exit?
(c.) (4%) What is the static pressure p_1 at the funnel inlet?
(d.) (6%) What force is required to tug the funnel?
(e.) (3%) What flow rate Q can be sustained through the funnel in a frictionless flow?



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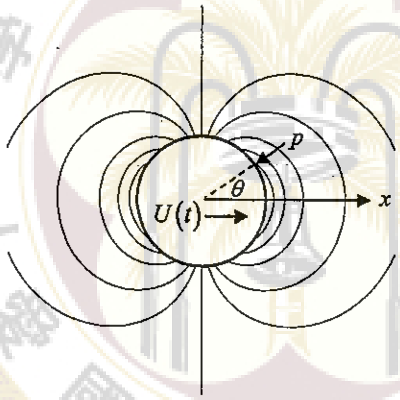
3. (25 %) Consider a cylinder of radius R moving at velocity $U(t)\hat{x}$ in the potential flow of a fluid with density ρ. Let the origin be at the cylinder center. The velocity potential is given as $\phi = A\cos\theta / r$, where A is the coefficient to be determined.

(a) Using the boundary condition: $\frac{\partial \phi}{\partial r} = U(t)\cos\theta$ on the cylinder surface to find the solution of ϕ. (5%)

(b) Using the unsteady Bernoulli's equation: $\frac{\partial \phi}{\partial t} + \frac{1}{2}|\nabla \phi|^2 + \frac{p}{\rho} = C(t)$ (where C is a function of time) and the velocity potential ϕ in (a) to find the pressure $p(\theta)$ on the cylinder surface. (5%)

(c) Integrate the surface pressure $p(\theta)$ in (b) to find the force (per unit length) f_x exerted on the cylinder by the fluid along the x direction: $f_x = -\int_0^{2\pi} p(\theta)\cos\theta R d\theta$. (10%)

(d) Based on (c), determine the "added mass" (per unit length) of the cylinder in the fluid. (5%)



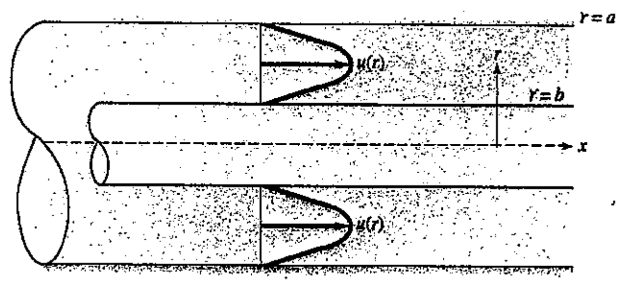
4. (25 %) Consider a steady axial laminar flow with the pressure drop $-\frac{dp}{dx} = K$ in the annular space between two concentric cylinders with inner radius b and outer radius a. Assume there is no slip at the surface and the velocity $u = u(r)$ only.

(a) State the governing equation and boundary conditions for the velocity $u(r)$. (7%)

(b) Based on (a), find the solution of $u(r)$ (8%).

(c) Calculate the volume flow rate Q (5%).

(d) Determine the position r' at which the velocity $u(r)$ attains its maximum (5%).



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