1. (19%) Given an arbitrary anti-symmetric 3×3 matrix

$$A=\begin{bmatrix} 0 & A_{12} & A_{13} \\ -A_{12} & 0 & A_{23} \\ -A_{13} & -A_{23} & 0 \end{bmatrix}$$, let the eigenvalues be λ_1, λ_2, and λ_3, and the corresponding *unit eigenvectors* (length=1) are $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$.

(a) (3%) Find the eigenvalues. If λ_1 is real, show λ_2 and λ_3 are a pair of complex conjugates, and $\mathbf{v}$ and $\mathbf{w}$ are also complex conjugate vectors.

(b) (4%) Let the eigenmatrix $Q=[\mathbf{u}\ \mathbf{v}\ \mathbf{w}]$. Show that $Q^T Q = I$.

(c) (6%) Let $\mathbf{v}=\mathbf{x}+i\mathbf{y}$, and matrix $P=[\mathbf{u}\ \mathbf{x}\ \mathbf{y}]$ ($\mathbf{u}$, $\mathbf{x}$, and $\mathbf{y}$ are column vectors). Show P is nonsingular. Find the matrix B such that $AP=PB$ and $A=PBP^{-1}$.

(d) (6%) Let $A_{12}=A_{13}=A_{23}=1$, compute λ_1, λ_2, and λ_3, and $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$.

2. (14%) Given an arbitrary non-symmetric matrix A, assume that A possesses real non-repeated eigenvalues: $\lambda_1, \lambda_2, \cdots, \lambda_n$. Let A^T denote the transpose of A. Let P and Q be the eigenmatricies of A and A^T.

(a) (4%) Show that $\lambda_1, \lambda_2, \cdots, \lambda_n$ are also eigenvalues of A^T.

(b) (4%) What is the relationship between P and Q?

(c) (6%) Use $\lambda_1, \lambda_2, \cdots, \lambda_n$ and the two eigenmatricies P and Q to solve the equation $Ax=b$ (where x and b are $n\times 1$ column vectors).

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3. (10%) Find the general solution of the differential equation
$x^3 y''' - 3x^2 y'' + 7xy' - 8y = x^2$

4. (25%) Governing equation for harmonic motion of a simply supported Euler beam of length L can be expressed as

$$\frac{d^4 y}{dx^4} + 16y = 1, \quad 0 < x < L$$

with boundary conditions $y(0)=0, y''(0)=0, y(L)=0, y''(L)=0$

(a) (15%) find the solution $y(x)$ when $L=1$.

(b) (10%) discuss also the solution (and the physical meaning) when $L=\pi$.

5. (32%) 1-D heat equation in a bar is shown as follows:

$$\frac{\partial T(x,t)}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2},$$

where $T(x, t)$ represents the spatial temperature distribution at time t and α is the thermal conductivity. Consider the case that this bar is perfectly insulated and also at two ends $x = 0$ and $x = L$ (adiabatic conditions) with initial temperature distribution $A(x,0)$.
[Hint: the heat flux through the faces at the ends of a bar is found to be proportional to $T_n = \partial T / \partial n$.]

(a) (8%) Define this Initial-boundary-value problem by using proper initial and boundary conditions.
(b) (12%) Use the separation of variables to find the general solution $T(x,t)$ by substituting boundary conditions
(c) (12%) Use the general solution found in (b), find the coefficient of the n-th component in the general solution $T(x,t)$ by substituting initial condition and using Fourier series.

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