

1. A cycloid can be parametrized by the functions $x(\theta) = (\theta - \sin \theta)$, $y(\theta) = (1 - \cos \theta)$.
 - (a) Find dy/dx and d^2y/dx^2 . (10%)
 - (b) Plot the cycloid curve. $0 \leq \theta \leq 2\pi$. (10%)
 - (c) Find the length of the cycloid arch. (10%)
 - (d) Find the surface area of the solid generated by revolving the cycloid arch about the x-axis. (10%)
2. Calculate $\int \sin^5 x dx$. (10%)
3. Test the convergence of the series $\sum \frac{1}{k} \left(\frac{1}{\ln k}\right)^{3/2}$. (10%)
4. Minimize $x^2 + y^2$ on the curve $x^4 + 7x^2y^2 + y^4 = 1$. (10%)
5. Integrate $\vec{h}(x, y) = (x + 2)y\vec{i} + (2x + y)\vec{j}$ over the indicated path $y = x^2$ from $(0, 0)$ to $(2, 4)$. (10%)
6. Take Ω as the parallelogram bounded by $x - y = 0$, $x - y = \pi$, $x + 2y = 0$, $x + 2y = \pi/2$. Evaluate $\iint_{\Omega} \sin(x - y) \cos(x + 2y) dx dy$. (10%)
7. Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} \frac{1}{x^2 + y^2 + z^2} dz dy dx$. (10%)

試題隨卷繳回