

1. Describe below in brief. (10%)
(a) blackbody radiation (b) Rydberg constant (c) pair annihilation (d) Lorentz transformation
(e) Kohn-Sham equation.

2. (10%) A π^0 meson at rest decays into two photons of equal energy. What is the wavelength of the photons? (The mass of the π^0 is $135 \text{ MeV}/c^2$)

3. (20%) Show that the energy E_n of positronium is given by $E_n = -\alpha^2 m_e c^2 / 4n^2$, where m_e is the electron mass, n is the principal quantum number, and α is the fine structure constant. Show that the radii are expanded to double the corresponding radii of hydrogen atom. Show that the transition energies are halved compared to that of hydrogen atom.

Hint: A positronium is a system consisting of an electron and its anti-particle, a positron, bound together into an "exotic atom".

4. (20%) Consider the transitions in heavy atoms which give rise to L_α line in X-ray spectra. Show the possible allowed transitions from which state to which state by using term symbols under the selection rules are $\Delta l = \pm 1$, $\Delta j = 0, \pm 1$.

Hint: The L_α line is produced due to transition $n = 3 \rightarrow n = 2$.

5. (20%) (a) Show that $[x, p_x] = [y, p_y] = [z, p_z] = i\hbar$
(b) Given that $L = r \times p$, show that $[L_x, L_y] = i\hbar L_z$

Hint: $p_x = -i\hbar \frac{\partial}{\partial x}$

6. (20%) A steady stream of particles with energy $E (> V_0)$ is incident on a potential step of height V_0 as shown in below figure. The wave functions in the two regions are given by
 $\psi_1(x) = A_0 \exp(ik_1x) + A \exp(-ik_1x)$
 $\psi_2(x) = B \exp(ik_2x)$

Write down expressions for the quantities k_1 and k_2 in terms of E and V_0 . Show that

$$A = \left[\frac{k_1 - k_2}{k_1 + k_2} \right] A_0 \text{ and } B = \left[\frac{2k_1}{k_1 + k_2} \right] A_0$$

and determine the reflection and transmission coefficients in terms of k_1 and k_2 . If $E = 4 V_0/3$ show that the reflection and transmission coefficients are $1/9$ and $8/9$ respectively. Also, comment on why $A^2 + B^2$ is not equal to 1.

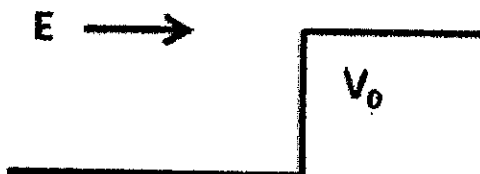


Figure. Potential step for Problem 6.