國立臺灣大學 103 學年度碩士班招生考試試題

1. (10%) Find the value of the determinant

$$\begin{vmatrix} 1 & \alpha & \alpha^2 & \alpha^3 \\ 1 & \beta & \beta^2 & \beta^3 \\ 1 & \gamma & \gamma^2 & \gamma^3 \\ 1 & \delta & \delta^2 & \delta^3 \end{vmatrix}$$

2. (10%)If a periodic function $f(x)$ can be expressed as its Fourier series, i.e.

$$f(x) = a_0 + \sum_{n=1}^{\infty}\left[a_n \cos n\omega_0 x + b_n \sin n\omega_0 x\right]$$

where $\omega_0 = \frac{2\pi}{T}$, T is the period of function $f(x)$

Prove that $\frac{1}{T}\int_{-T/2}^{T/2} f^2(x)\,dx = a_0^2 + \frac{1}{2}\sum_{n=1}^{\infty}\left[a_n^2 + b_n^2\right]$

3. (20%) According to Newton's law of gravity, the potential between two particles at points $P_0:(x_0, y_0, z_0)$ and $P:(x, y, z)$ can be expressed as $f(x,y,z) = c/r$, where c is a constant and r is the distance between P_0 and P. Prove that the potential function $f(x,y,z)$ is a solution of Laplace equation.

4. (20%) Find the general solution of the non-homogeneous differential equation

$$\left(D^2 + 6D + 9I\right)y = 16e^{-3x} \Big/ \left(x^2+1\right)$$

5. (20%) Let λ be an eigenvalue of the unitary matrix **U**. Then prove that $|\lambda| = 1$.

6. (20%) Solve the one dimensional wave equation with the initial conditions,

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}, \qquad -\infty < x < \infty$$

$$\text{I.C.s} \quad \begin{cases} u(x,0) = f(x) \\ u_t(x,0) = g(x) \end{cases}, \qquad -\infty < x < \infty$$

試題隨卷繳回