# 國立臺灣大學 103 學年度碩士班招生考試試題



1. (30%) Consider a two-dimensional parallel plate microfluidic flow channel of thickness $h$ and depth $w$ (into the paper) connecting two liquid reservoirs on the respective right and left hand sides of the flow channel. Liquid water of viscosity $\mu$ and density $\rho$ is poured into the left reservoir, and by capillary pumping, a liquid column starts to grow within the microchannel. In this problem, we would like to investigate the growth of this liquid column (or advancement of the liquid meniscus) with respect to time, i.e., $L\,(t)$, by taking the following steps:

(i) (5%) Considering hydrostatic forces, please evaluate the pressure $p_1$.

(ii) (5%) Assume constant values for the surface tension, $\gamma$, and the contact angle, $\theta$, near the meniscus (or at the interface of the water and microchannel walls) of the liquid column inside the microchannel. Please calculate $p_2$ by performing a force balance on the liquid meniscus.

(iii) (10%) Given the pressure difference of $p_1 - p_2$, please calculate the flow field as well as the volume flow rate within the liquid column or two-dimensional parallel plate micro flow channel.

(iv) (10%) Using a control volume analysis, please derive a differential equation that governs the growth of the liquid column, i.e., $L\,(t)$, inside the microchannel. Solve this equation when subjected to the initial condition of $L = 0$.

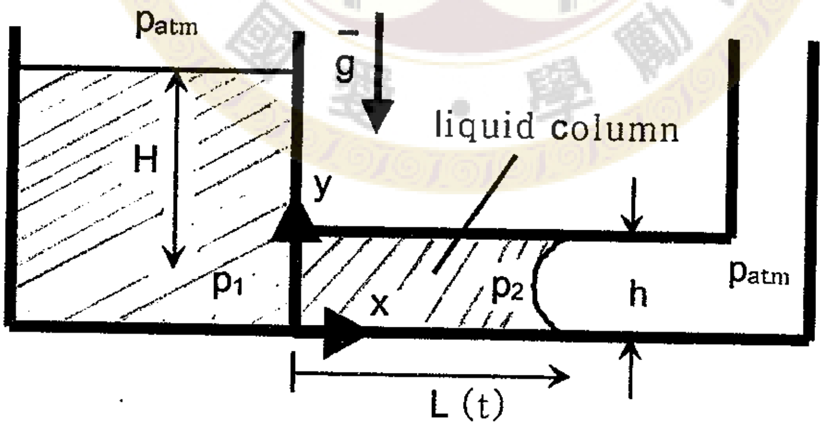


**Fig. 1.** Capillary pumping inside a two-dimensional parallel plate microfluidic flow channel.

2. (20%) A fluid, driven by pressure, flows in the positive $x$-direction within a long flat duct of unit depth (into the paper) with the length and thickness of the duct being respectively given by $L$ and $h$ ($h \ll L$). The duct has porous walls such that injection of the fluid into the duct is achieve at $y = 0$ and that suction of the fluid from the duct is obtained at $y = h$. As a result, a constant cross flow is maintained everywhere within the duct, that is, $u_y = v_0$ with $v_0$ being a constant. Neglecting gravitational effects and assuming constant fluid properties (e.g., viscosity, $\mu$, density, $\rho$, etc.), please solve for the velocity distribution, $u_x$, as well as the mass flow rate, $\dot{m}$, in the $x$-direction within the long flat duct under steady and fully developed conditions.

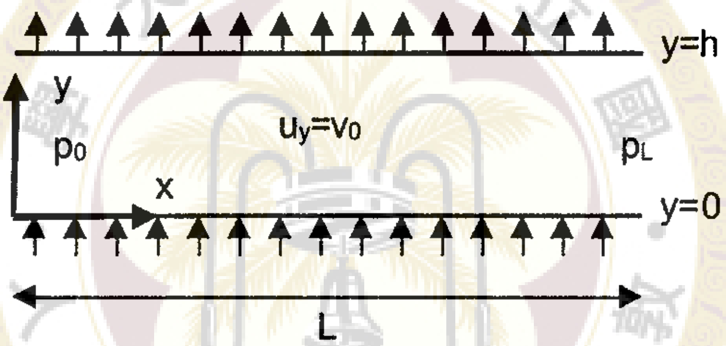


**Fig. 2.** Parallel plate duct flow subjected to suction and injection of fluid at the wall boundaries.

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3. 山坡地的泥土受雨水沖刷而不斷流失，甚至造成土石流的問題，一直是近年來備受關注，非常嚴重的環境事件。

現在讓我們考慮山坡地泥土流失的情況，如附圖所示。第一種情況是長期受雨水浸泡的影响，山坡地在重力的作用下開始往下滑動；第二種情況則是泥土上方同時受到山頂累積的大量雨水向下流動造成的沖刷效應。為方便分析，假設：

(1) 受雨水浸泡過的泥土，或稱為泥漿，在性質上如同 "ideal plastic fluid"，也就是當受到剪應力，$\tau$，的作用時，其流動而剪應力的關係為 $\tau=\tau_y+\mu_{mud}\frac{du}{dy}$，其中 $\tau_y$ 為 yielding stress；亦即當受到小於 $\tau_y$ 的剪應力時，泥漿並不會流動，形同固体。此外，$\rho_{mud}$ 和 $\mu_{mud}$ 分別為泥漿的密度和黏性係數。

(2) 泥漿厚度為 H，它的下層可視為乾硬不會滑動的泥土層。

(3) 當雨水自山頂流下時，其流動速度為 U，且不考慮正在下雨的情況。

(4) 為簡化分析，當雨水自山頂流下時，其接近泥漿的速度分佈為 $u_w(y)=U(1-\exp(-\frac{V}{\nu_w}(y-H)))$，其中 V 為雨水流下時滲入泥漿的小量流速，$\nu_w$ 為雨水的 kinematic viscosity。

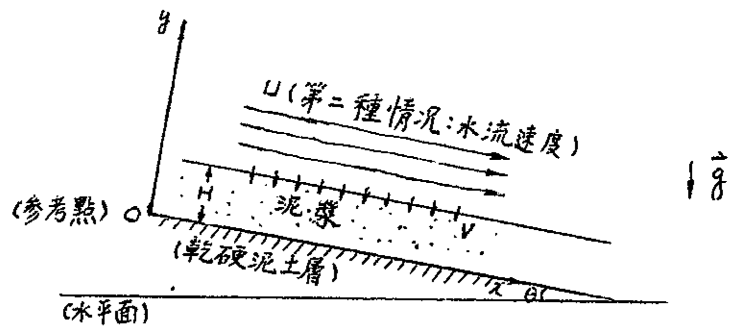
請根據以上的假設，求解下列的問題：(考慮第一種及第二種情況)

(10%) (1) 什麼是山坡地的臨界斜度 $\theta_{cr}$？也就是當山坡地的斜度 $\theta$ 大於 $\theta_{cr}$ 時，泥漿才會開始向下滑動。

(20%) (2) 當 $\theta>\theta_{cr}$ 時，請求出泥漿的速度分佈，$u_{mud}(y)$，並定性畫出其分佈圖。

(15%) (3) 泥漿下滑的總流失率 (volume flow rate)，Q。

(5%) (4) 什麼是此問題的重要無因次參數？




附 圖

試題隨卷繳回