1. (25%) First draw free-body diagrams and derive equilibrium equations. Then draw deformation diagrams and deduce the corresponding kinematic relations.

   (a) (10%) Draw a free-body diagram of a differential element of length dx, indicate all forces/moments involved, and on the basis of the drawing derive the formula relating the torsional moment $T(x)$ to a distributed torsional moment load $t(x)$. Then draw a twisting deformation diagram and deduce the corresponding relation between the twisting angle $\phi(x)$ and shear strain $\gamma(x, r)$, where r is the position radius.

   (b) (15%) Draw a free-body diagram of a differential element of length dx, indicate all forces/moments involved, and on the basis of the drawing derive the formulae which relate the bending moment $M(x)$ to the shear force $V(x)$, and $V(x)$ to a distributed transverse load $q(x)$. Then draw a flexural deformation diagram and deduce the corresponding relations of the deflection $w(x)$, curvature $\kappa(x)$, and normal strain $\epsilon(x, z)$, as well as shear strain $\gamma(x, z)$.

2. (25%) A horizontal bar (shaft/beam) made of a material with Young's modulus $E =$ 2.6 GPa and the shear modulus $G = 1$ GPa has a length $L = 10$ m and circular cross-section with diameter 1 m. The bar is fixed at both ends and is subjected to a concentrated vertical downward load $P = 31$ kN at the point $x = 4$ m. The line of action of P has an eccentricity $e = +10$ cm.

   (a) (10%) Find the reactions at the the left end $x = 0$ m.

   (b) (15%) Determine the distributions of (b1) normal stress, (b2) flexural shear stress, and (b3) torsional shear stress on the cross section at the left end $x = 0$ m.

3. (20%) A horizontal beam of length $L = 10$ m, height $h = 100$ cm, width $b = 40$ cm is made of a material with Young's modulus $E = 200$ GPa and thermal expansion coefficient $\alpha = 10 \times 10^{-6}/^{o}$C. It is fixed at the left end $x = 0$, whereas it's boundary condition at the right end $x = 10$ m may be fixed or may be free to move and bend. Now the temperature is raised up by $\Delta T = 40^{o}$C on the upper surface $z = -50$ cm. by merely $\Delta T = 20^{o}$C on the lower surface $z = +50$ cm, and varies linearly in between.

   (a) (8%) What amount of elongation (or contraction) u and deflection w at the right end does the beam undergo?

   (b) (8%) Or what thermal effects of axial (compressive or tensile?) force and bending (positive or negative?) moment are developed in the beam?

   (c) (4%) What is the key factor which decides the answer (a) or (b)?

4. (30%) A horizontal cantilever bar (rod/beam/column) made of a material with Young's modulus $E = 100$ GPa has a length $L = 2$ m and a thin-walled equal-leg angle section with each leg 20 cm long and 1.5 cm thick. The angle section is placed such that one leg is horizontal and the other leg is vertical upwards. The bar is fixed at the left end $x = 0$ m and is subjected to a horizontal load P at the right end $x = 20$ m on the intersection point of the two legs.

   (a) (6%) Locate the shear center and the centroid of the angle section.

   (b) (10%) When P is *tensile* and $P = 1$ kN, draw the axial force diagram and the bending moment diagram(s).

   (c) (14%) When P is *compressive*, derive the formula and determine the buckling load.

試題隨卷繳回