1. (50 points) Consider a household who lives for three periods, $t = 0, 1, 2$, and values consumption in each period, c_0, c_1, c_2. Each period, he receives incomes, y_0, y_1, y_2, which are exogenous given. Suppose he can choose two kinds of bonds, one-period (short term) bond, b_t and two-period (long term) bond, x_t, which pays interest rates, r^b, one period later, and r^x, two periods later, respectively. Suppose the initial bond holding, b_0, x_{-1}, x_0, and b_3, x_2, x_3 all equal zero. The optimal three-period problem of the household is

$$\begin{aligned} \max U(c_0, c_1, c_2) &= u(c_0) + \beta u(c_1) + \beta^2 u(c_2) \\ \text{s.t.} \quad c_0 + b_1 + x_1 &\leq y_0 \\ c_1 + b_2 &\leq y_1 + b_1(1 + r_1^b) \\ c_2 &\leq y_2 + b_2(1 + r_2^b) + x_1(1 + r_1^x). \end{aligned}$$

(a) Please derive the optimal conditions for $c_0, c_1, c_2, b_1, b_2, x_1$ and describe the economic meaning of each equation.

(b) Please derive the relationship between the interests of one-period bond and two-period bond, and describe its economic meaning.

(c) Suppose the utility function is $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$. Please compute the equilibrium interest rates r_1^b, r_2^b, r_1^x.

(d) Suppose there is an increase for the period zero income, y_0, what happen to the equilibrium interest rates? Please explain your findings.

(e) If the income now grows at a constant rate, g, i.e., $y_2 = (1+g)y_1 = (1+g)^2 y_0$. Suppose there is an increase for the growth rate, g, what happen to the equilibrium interest rates? Please explain your findings.

2. (30 points) Consider a two-period model, $t = 1, 2$, in which the utility function of a consumer is given by $U = \ln(c_1) + \beta \ln(c_2)$. A consumer is endowed with an asset (land or house) k at date 1 and consumption good y at date 2. The price of asset in terms of consumption good at date 1 is q_1. Suppose there is a credit market at date 1 in which consumers can borrow or lend. Let b_1 is date 1 borrowing and R be the gross interest rate.

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(a) Suppose the agent can borrow and lend freely. Outline the maximization problem and solve for the optimal consumption plan. If $\beta = 0.5$, $q_1 = 1$, $k = 2$, $y = 4$, and $R = 1$. What is the date 1 borrowing b_1 to achieve the optimal consumption plan?

(b) Suppose the agent faces a borrowing constraint $b_1 \leq \theta q_1 k$, where $0 < \theta < 1$. Explain this borrowing constraint. If $\theta = 0.6$, solve for consumption c_1 and c_2. How does the credit constraint affect the consumer's welfare?

(c) How does a rise in asset price from $q_1 = 1$ to $q_1 = 2$ affect the consumer's consumption plan? Explain.

3. (10 points) Suppose we estimate the following reaction function of a central bank:

$$R_t = \alpha_0 + \alpha_1(\pi_t - \pi^*) + \alpha_2(y_t - y^*) + \alpha_3 z_t, \tag{1}$$

where R_t is the short-term interest rate, π_t and y_t are respectively date t inflation rate and output growth rate, and z_t is date t exchange rate (NT\$/US\$). Explain the idea why we estimate a reaction function of central bank such as equation (1). Suppose the estimation results are $\alpha_1 = 2.0$ and $\alpha_3 = 0$ for country A and $\alpha_1 = 0.8$ and $\alpha_3 = 0.2$ for country B. Explain the differences between the behavior of these two central banks.

4. (10 points) Suppose domestic borrowers borrow foreign debt in terms of US dollars. Explain what will happen to the balance sheet of domestic borrowers when there is an unexpected depreciation in exchange rate.

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