國立臺灣大學 103 學年度碩士班招生考試試題

Entrance Examination

1. (15%) Let X_1, X_2 and X_3 be a random sample from a $Poisson(\lambda)$. Moreover, let $Y_1 = X_1 + X_3$, $Y_2 = X_2 + X_3$, and $Z_i = 1_{(Y_i=0)}$, $i = 1, 2$. Compute the correlation of Z_1 and Z_2.

2. (5%) (5%) Let $(X, Y)^T$ be a bivariate random vector with finite variances. Show that $Cov(X, Y - E[Y|X]) = 0$ and $Var(Y - E[Y|X]) = E[Var(Y|X)]$.

3. (15%) Let $(X_1, Y_1), \dots (X_n, Y_n)$ be a random sample from a bivariate normal distribution with correlation coefficient ρ. Using the fact that $\sqrt{n}(r - \rho) \xrightarrow{d} N(0, (1 - \rho^2)^2)$, where r is the sample correlation coefficient, try to find a statistic $g(r)$ which converges to a normal distribution with constant variance.

4. (15%) Let $X_1, \dots, X_n$ be a random sample from a population with a probability density function

$$f_X(x|\theta) = \theta x^{\theta-1}, \quad 0 < x < 1, \quad 0 < \theta < \infty.$$

Find the the uniformly minimum variance unbiased estimator (UMVUE) of θ.

5. (15%) Let $X_1, \dots X_n$ be a random sample from $N(\theta, \sigma^2)$. Find an unbiased size α test for the null hypothesis $H_0 : \theta_1 \le \theta \le \theta_2$ versus the alternative hypothesis $H_A : \theta < \theta_1$ or $\theta > \theta_2$.

6. (15%) Let $X_1, \dots, X_n$ be a random sample from $Beta(\theta, 1)$ and θ have a $Gamma(\alpha, \beta)$ distribution, where α and β are known positive constants. Find the Bayes estimator of θ under the squared error loss function.

7. (15%) Let $X \sim f(x)$ and generate a random sample $Y_1, \dots, Y_n$ from $g(y)$, which has the same support of $f(x)$. Moreover, let $P(X^* = Y_k) = q_k$ with $q_k = (f(Y_k)/g(Y_k))/(\sum_{i=1}^n f(Y_i)/g(Y_i))$, $k = 1, \dots, n$. Show that $P(X^* \le x) \xrightarrow{p} P(X \le x)$ as $n \to \infty$.

試題隨卷繳回