1. (10 pts) Let $w = f(x, y)$ be a function of two variables, and let $x = u+v$ and $y = u-v$. Determine a and b, $a, b \in R$, such that
$$\frac{\partial^2 w}{\partial u \partial v} = a\frac{\partial^2 w}{\partial x^2} + b\frac{\partial^2 w}{\partial y^2}.$$

2. (15 pts) Let A be the closed and bounded region defined by $-1 \le x, y \le 1$, and let $f : A \to R$ be given by
$$f(x, y) = x^2y - 2xy.$$
Find the points in A where f attains a global maximum and a global minimum.

3. (10 pts) Find the critical points for the function
$$f(x, y) = 3x^2 + 2xy + 2x + y^2 + y + 4$$
and determine whether they are local maxima, local minima, or saddle points.

4. (10 pts) Compute the length ℓ of the polar curve whose polar model is
$$r(\theta) = 2\cos(\theta), 0 \le \theta \le \pi.$$

5. (15 pts) Calculate the integral $\int_{R^2} \exp(-3x^2 + 4xy - 3y^2)dxdy$.

6. (10 pts) Assume that f is differentiable at a. Evaluate
$$\lim_{x\to a} \frac{a^n f(x) - x^n f(a)}{x - a} \quad \text{where } n \text{ is a natural number.}$$

7. (10 pts) Determine whether or not the following limit exists, and find its value if it exists:
$$\lim_{n\to\infty} \left[n - \frac{n}{e}\left(1 - \frac{1}{n}\right)^n\right].$$

8. (20 pts) Let C be the curve $x^2+y^2 = 1$ lying in the plane $z = 1$. Let $\mathbf{F} = (z-y)\mathbf{i}+y\mathbf{k}$.
   - **(a)** (5 pts) Calculate $\nabla \times \mathbf{F}$.
   - **(b)** (5 pts)Calculate $\int_C \mathbf{F} \cdot ds$ using a parametrization of C and a chosen orientation for C.
   - **(c)** (5 pts) Write $C = \partial S$ for a suitably chosen surface S and, applying Stokes' theorem, verify your answer in (b).
   - **(d)** (5 pts) Consider the sphere with radius $\sqrt{2}$ and center the origin. Let S' be the part of the sphere that is above the curve (i.e., lies in the region $z \ge 1$), and has C as boundary. Evaluate the surface integral of $\nabla \times \mathbf{F}$ over S'. Specify the orientation you are using for S'.