

1. (20 pts) Solve the equations.

(a) $y' = \frac{2x^3 + y^3}{x^2y}$.

(b) $y' = \frac{3xy + x + x^3}{1 + x^2}$.

2. (20 pts) Find the general solution of the system

$$\mathbf{x}'(t) = \begin{pmatrix} 5 & 1 \\ -1 & 7 \end{pmatrix} \mathbf{x}(t) + \begin{pmatrix} \cos t \\ 0 \end{pmatrix}.$$

3. (20 pts) Find the solution of the equation

$$\begin{cases} x^2y'' + 3xy' - 3y = 2 \ln x, \\ y(2) = 0, y'(2) = 1. \end{cases}$$

4. (20 pts) Suppose that $y'(x)$ is continuous for $x \in \mathbb{R}$. Find all solutions of

$$(y')^2 + (e^x - x - 1)y' - x(e^x - 1) = 0.$$

5. (20 pts) Let

$$\phi_0(t) = 0, \phi_{n+1}(t) = \int_0^t s\phi_n(s) ds + 1, n = 0, 1, 2, 3, \dots$$

(a) Show that $0 \leq \phi_n(t) \leq 2$ for $0 \leq t \leq 1$.

(b) Show that $\lim_{n \rightarrow \infty} \phi_n(t)$ exists for $0 \leq t \leq 1$.

(c) Find $\lim_{n \rightarrow \infty} \phi_n(t)$ for $0 \leq t \leq 1$.

試題隨卷繳回