題號： 55
科目：常微分方程
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國立臺灣大學 103 學年度碩士班招生考試試題

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1. (20 pts) Solve the equations.
(a) $y' = \dfrac{x^3 + 3y^3}{x^2 y}$.
(b) $y' = \dfrac{3xy - x - x^3}{1 + x^2}$.

2. (20 pts) Find the general solution of the system

$$\mathbf{x}'(t) = \begin{pmatrix} 2 & 1 \\ -1 & 4 \end{pmatrix} \mathbf{x}(t) + \begin{pmatrix} \sin t \\ 0 \end{pmatrix}.$$

3. (20 pts) Find the solution of the equation

$$\begin{cases} x^2 y'' + 3xy' - 3y = 5 \ln x, \\ y(2) = 0, \ y'(2) = 1. \end{cases}$$

4. (20 pts) Supppose that $y'(x)$ is continous for $x \in \mathbb{R}$. Find all solutions of

$$(y')^2 + (e^x - x - 1)y' - x(e^x - 1) = 0.$$

5. (20 pts) Let

$$\phi_0(t) = 0, \ \phi_{n+1}(t) = \int_0^t s \phi_n(s)\, ds + 2, \ n = 0, 1, 2, 3, \cdots.$$

(a) Show that $0 \le \phi_n(t) \le 4$ for $0 \le t \le 1$.
(b) Show that $\lim_{n \to \infty} \phi_n(t)$ exists for $0 \le t \le 1$.
(c) Find $\lim_{n \to \infty} \phi_n(t)$ for $0 \le t \le 1$.

試題隨卷繳回