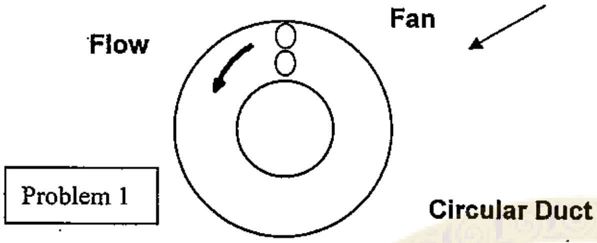
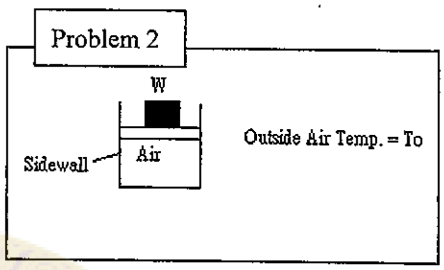
國立臺灣大學 102 學年度碩士班招生考試試題

1. (20%) Consider the flow of air driven by a fan inside a round circular duct, with circular cross-section, as shown below. Before the fan starts, the temperature inside the duct is the same as that of outside the duct. The duct wall is adiabatic.
   (a) After the fan is turned, describe the temperature inside the duct.
   (b) Write down the differential equation and boundary conditions to solve for the temperature inside. (Do not solve the equation.)





2. (30%) The ideal gas law is P= ρRT, where P is the pressure, ρ the gas density, R the gas constant, T the temperature. A piston, with frictionless and massless top plate of area A, is pressed down by a weight W (see Figure). Inside the piston is air with mass M, volume V, and temperature T. The outside air has a constant temperature T_0. The initial state of air inside is P_1, ρ_i, T_1.

a) Consider the situation when the sidewall is <u>NOT insulated</u>, i.e., heat can flow freely through the side-wall. If the weight is increased twice, i.e., to 2W, what is the final volume?

b) For the case of a), what is the final temperature?

c) Starting from the initial state, consider the entire system (sidewall and top moveable piston) are all now <u>well insulated</u>, with initial air temperature of T= T_0, if the weight is increased twice, i.e., to 2W, what is the final volume? (Hint: you need another thermodynamic principle here.)

d) For the case of c), what is the final temperature?

3. A stationary power plant operating on an ideal Brayton cycle has a pressure ratio of 8. The working fluid is air, which behaves as an ideal gas. For simplicity, constant specific heats are assumed for air. The gas temperature is $300K$ at the compressor inlet and $1300K$ at the turbine outlet.
(a) Draw a $T-s$ diagram for the Brayton cycle and state your assumptions. (8%)
Assume constant specific heats, find out
(b) the gas temperature at the exits of the compressor and the turbine (6%),
(c) the back work ratio (5%),
(d) the thermal efficiency (6%).

Notes: The gas constant of air is $R = 0.287kJ/(kg \cdot K)$, and its specific heats at room temperature $(300K)$ are $C_p = 1.005kJ/(kg \cdot K)$.

4. The Clausius-Clapeyron equation describes the variation of boiling point with pressure. It is usually derived from the condition that the chemical potentials of the gas and liquid phases are the same at coexistence. For an alternative derivation, consider a Carnot engine using one mole of water. At the source (P,T) the latent heat L is supplied converting water into stream. There is a volume increase V associated with this process. The pressure is adiabatically decreased to $P-dP$. At the sink $(P-dP, T-dT)$ stream is condensed back to water.
(a) Show that the work output of the engine is $W = VdP + O(dP^2)$ and find out the Clausius-Clapeyron equation in this case (8%).
(b) Assume that L is approximately temperature-independent, and that the volume change is dominated by the volume of stream treated as an ideal gas, that is, $V = Nk_BT/P$. Find $P(T)$ (8%).
(c) A typhoon works somewhat like the engine described above. Water evaporates at the warm surface of the ocean, stream rises up in the atmosphere, and condenses to water at the higher and cooler altitudes. The Coriolis force converts the upward suction of the air to spiral motion. Typical values of warm ocean surface and high altitude temperatures are $300K$ and $180K$, respectively. The warm water surafce layer must be at least 100 meters thick to provide sufficient water per hour to maintain itself. Estimate the maximum possible efficiency, and power output, of such a typhoon. (The latent heat of vaporization of water is about $2.3 \times 10^6 Jkg^{-1}$.) (9%)

試題隨卷繳回