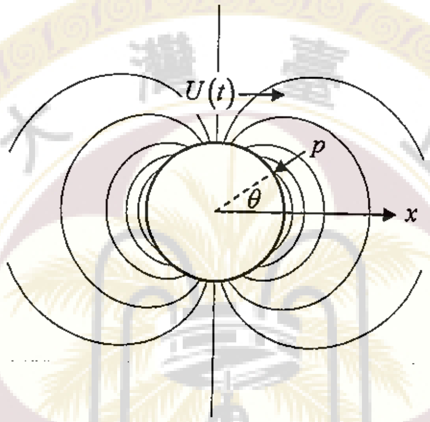
1. (25 %) Consider a sphere of radius R moving at velocity $U(t)\hat{x}$ in the potential flow of a fluid with density ρ. The velocity potential is given as $\phi = A\cos\theta / r^2$, where A is the coefficient to be determined.

(a) Using the boundary condition on the sphere to find the solution of ϕ. (5%)

(b) Using the unsteady Bernoulli's equation: $\frac{\partial \phi}{\partial t}+\frac{1}{2}u^2+\frac{p}{\rho}=C$ (where u is the velocity and C is a constant in space) and ϕ in (a) to find the pressure p on the sphere surface. (5%)

(c) Integrate the surface pressure in (b) to find the force exerted on the sphere by the fluid: $F_x = \int -p\cos\theta dA$. (10%)

(d) Based on (c), determine the "added mass" of the sphere in the fluid. (5%)

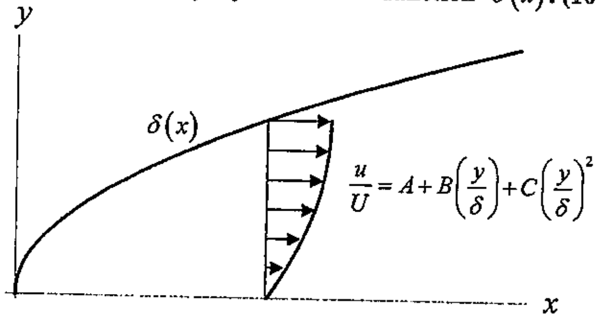


2. (25 %) The velocity profile of the laminar boundary layer in a fluid with density ρ and viscosity μ is given as $\frac{u}{U}=A+B\left(\frac{y}{\delta}\right)+C\left(\frac{y}{\delta}\right)^2$, where U is the free stream velocity, δ is the boundary layer thickness, and A, B, and C are coefficients to be determined.

(a) Using the boundary conditions to find the solution of u. (5%)

(b) Bases on (a), find the displacement thickness: $\delta^* = \int_0^\delta \left(1-\frac{u}{U}\right)dy$ and momentum thickness: $\theta = \int_0^\delta \frac{u}{U}\left(1-\frac{u}{U}\right)dy$. (10%)

(c) Using the von Kármán momentum equation: $\tau_0 = \rho U^2 \frac{d\theta}{dx}$ (where $\tau_0 = \mu \left.\frac{du}{dy}\right|_{y=0}$ is the wall shear stress) and θ in (b) to find the boundary layer thickness function $\delta(x)$. (10%)



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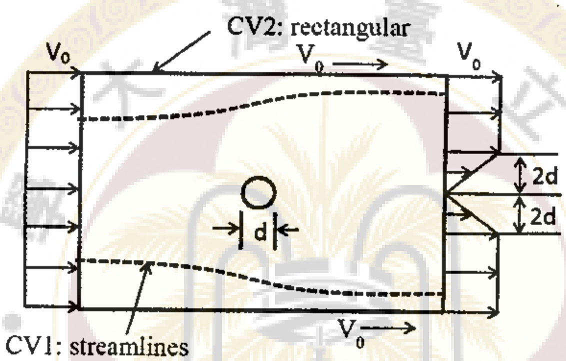
3. (25%) In an experiment to determine drag, a circular cylinder of diameter d was immersed in a steady, two-dimensional, unconfined incompressible flow with density ρ. Measurement of velocity and pressure were made at the boundaries of the fixed control volume, CV1 and CV2 shown below. CV1 has sides that are streamline surfaces. CV2 is rectangular. The pressure was uniform over the control surface. The x-component of velocity was as indicated by the sketch.

(a)(10%) Find the drag exerted on the circular cylinder from the control volume analysis using CV1.

(b)(10%) Write down mass and momentum balance for CV2. Is it the same as what you had in (a)? If not, what is the difference?

(c)(5%) Obtain the drag exerted on the circular cylinder using CV2.



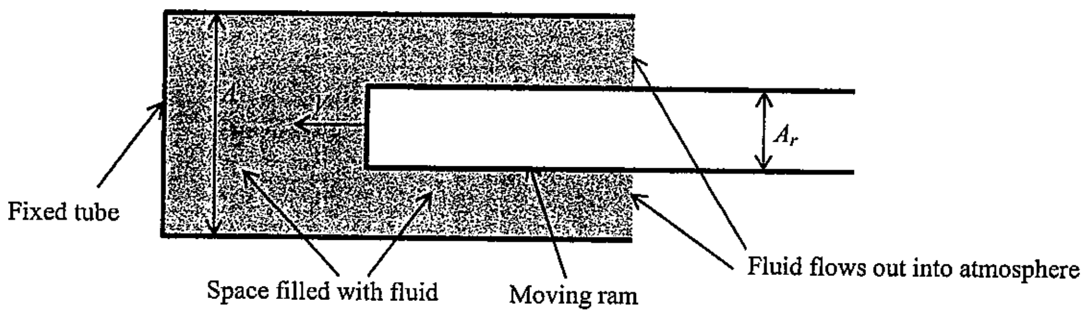
4. (25%) A hydraulic ram in the sketch below is used to generate 'resistive force' in an industrial machine. The ram has an area A_r and pushes against the hydraulic fluid contained in a fixed tube of area A. The fluid flows out over the ram into the atmosphere. Suppose that the ram is moving at a constant speed V. The area of the ram A_r is smaller than the tube area A, but the fluid layer between the ram and the tube is sufficiently thick to ignore viscous effects.

(a)(5%) What is the velocity of the fluid being forced out?

(b)(10%) What is the pressure on the vertical face of the ram?

(c)(5%) What resistive force is generated to the ram?

(d)(5%) Sketch the *actual* velocity profile you would expect in the fluid layer between the ram and tube. What do you expect your inviscid analysis of part(b) to fail?



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